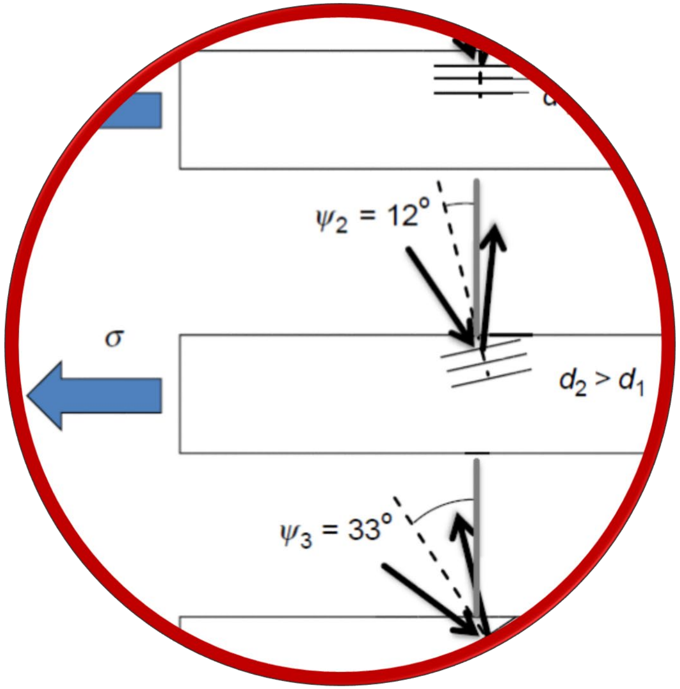
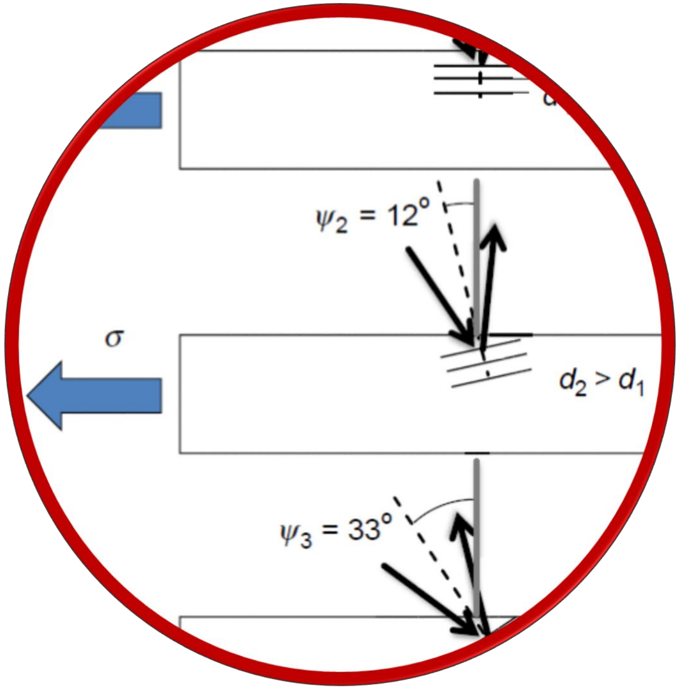
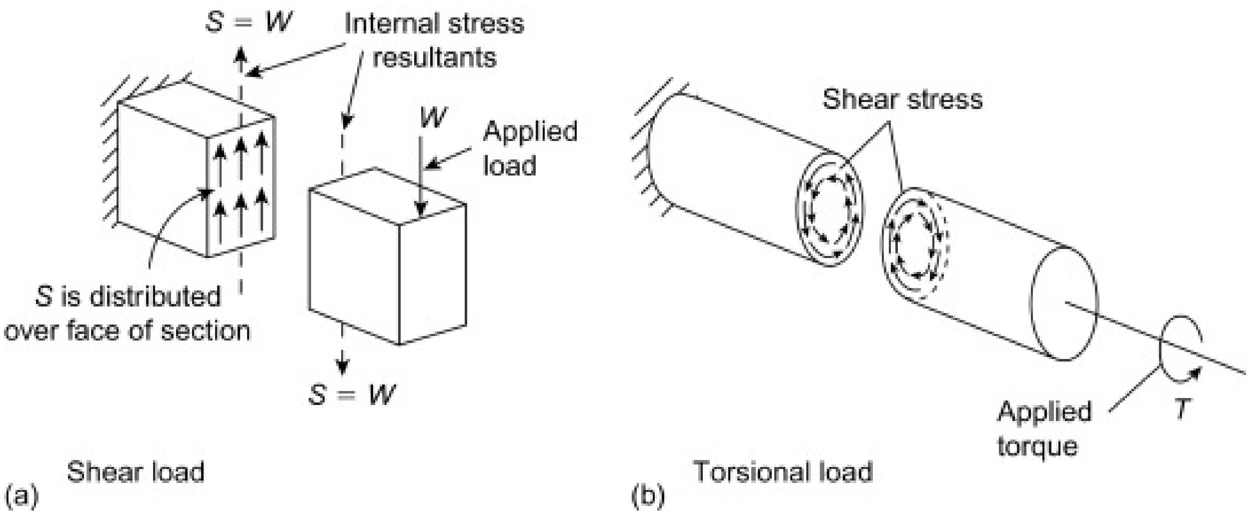
3. Materials mechanics



Stresses and strains

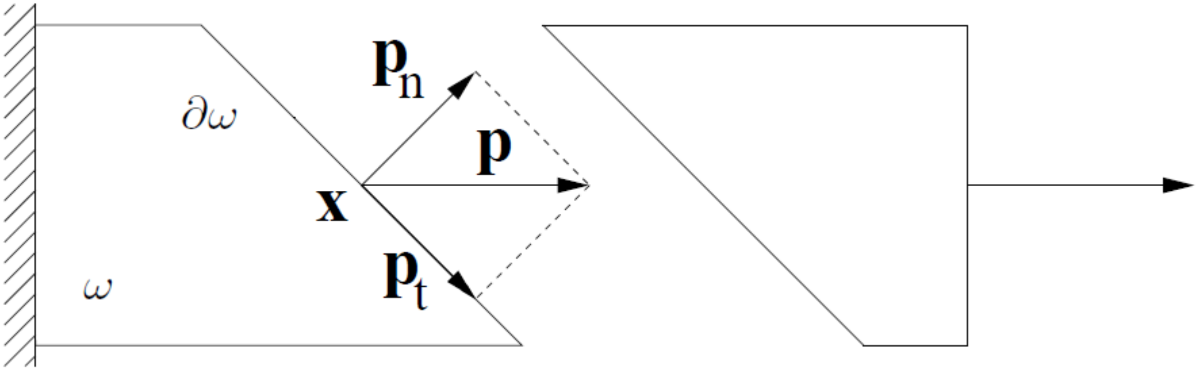


External and internal forces



A set of external forces exerted on a solid induce internal cohesive forces, which are the origin of stresses responsible for deformations of the material

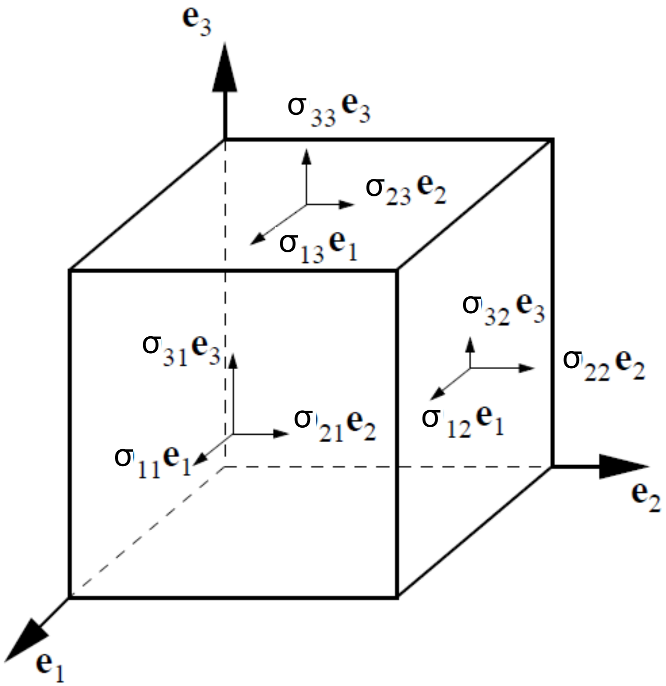
Normal and tangential decomposition



If a surface at an angle to the loading is observed or the loading is multiaxial, a decomposition into normal and tangential component of the total internal force can be benefitial.

Stress tensor

- Stress is defined as the force across an infitesimally small boundary, for all orientations of the boundary.
- It is represented by a second order stress tensor
- The component ij of the tensor corresponds to the component i of the internal force by unit surface of orientation j.



$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

Hydrostatic and deviatoric stresses

The stress tensor can also be decomposed into a hydrostatic and a deviatoric part:

$$\mathbf{S} = \frac{1}{3} tr(\mathbf{S})\mathbf{I} + \mathbf{S}' = -p\mathbf{I} + \mathbf{S}'$$

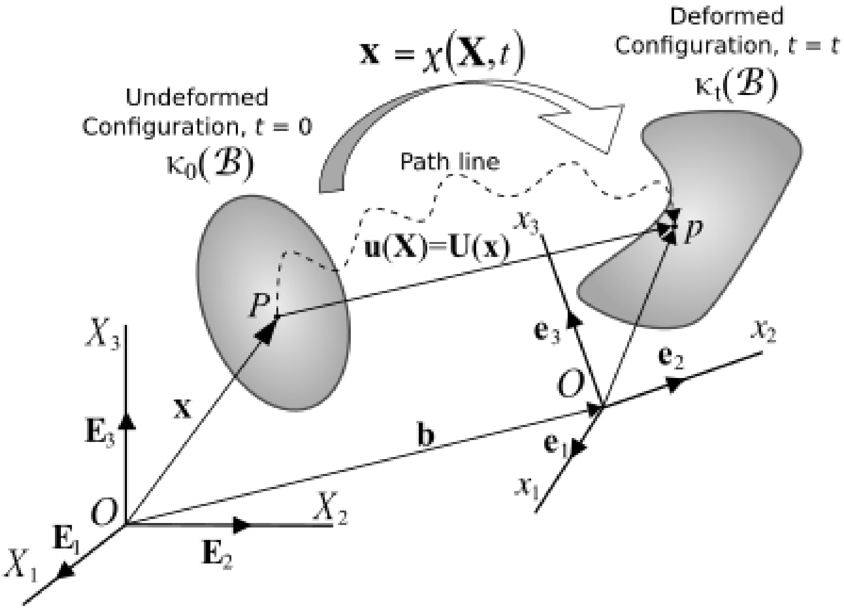
where p is hydrostatic pressure and S' the stress deviator.

$$p = -\tfrac{1}{3} tr(\mathbf{S})$$

The trace is defined as $tr(\mathbf{S}) = \sigma_{11} + \sigma_{22} + \sigma_{33}$. The definition of the stress deviator is

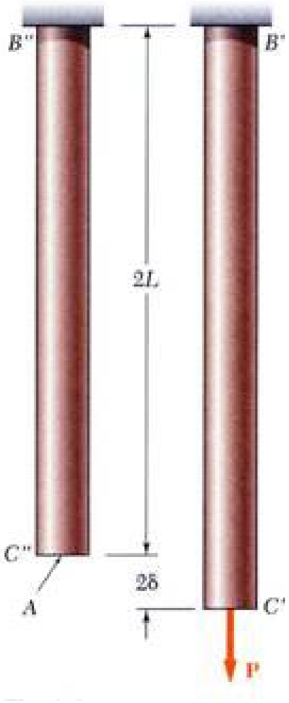
$$\mathbf{S}' = \mathbf{S} - \frac{1}{3} tr(\mathbf{S})\mathbf{I}$$

Strain



Strain is a measure of the material's deformation as a reaction to an applied load. Analogous to stress, it is represented by a second order tensor **ε**.

Normal strain

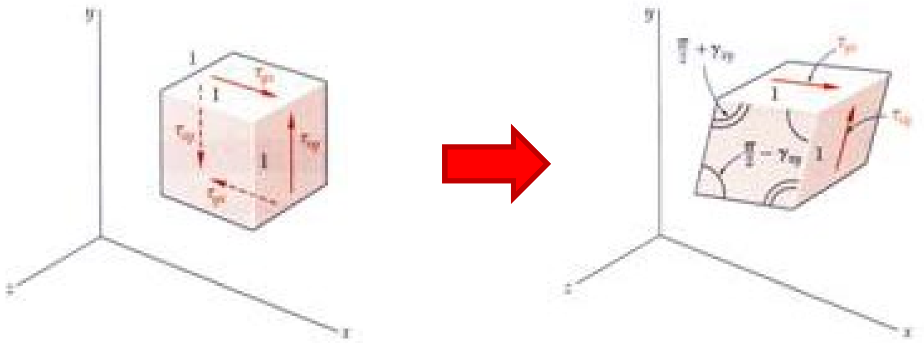


$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{2\delta}{2L} = \frac{\delta}{L}$$

Normal strains are length-changing stretches of an object resulting from a normal stress.

Shear strain



Shear strains are deformations changing the angles between neigbouring edges of an infinitesimally small cubic volume. They do not lead to a change in volume.

General definitions and tensor notation

For small rotations, infinitesimal normal strains are defined as:

$$\varepsilon_{ii} = \frac{\partial u_i}{\partial i} \qquad \text{for i } = 1{,}2{,}3$$

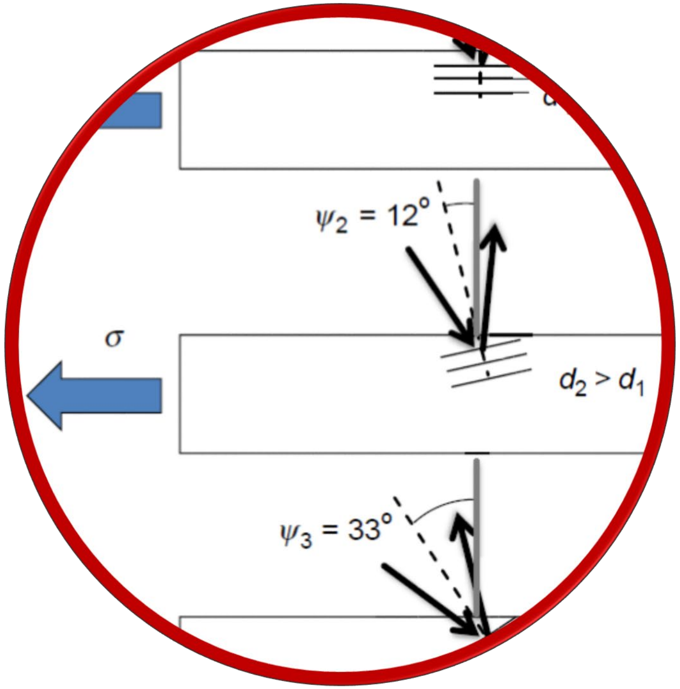
Shear strains are defined as:

$$\gamma_{ij} = \frac{\partial u_i}{\partial j} + \frac{\partial u_j}{\partial i}$$

The second order strain tensor takes the form:

$$\boldsymbol{\varepsilon}_{ii} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix}$$

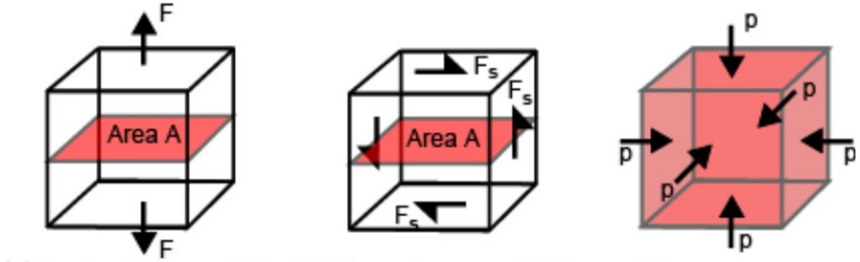
Elasticity



Elasticity

Definition:
Young's modulus,
shear modulus,
bulk modulus

Range:
0.1 MPa (foams) to 1000GPa (diamond)



(a) Tensile stress $\sigma = F/A$ usual units MPa (b) Shear stress $\sigma_s = F_s/A$ usual units MPa (c) Pressure p usual units MPa

Figure 2. (a) Tensile stress,. (b) Shear stress. (c) Hydrostatic pressure

Origin:

Bond type	Examples	Bond Stiffness S (N/m)	Young's Modulus E (GPa)
Covalent	**Carbon-carbon**	**50 - 180**	**200 - 1000**
Metallic	**All metals**	**15 - 75**	**60 - 400**
Ionic	**NaCl**	**8 - 24**	**32 - 96**
Hydrogen bond	**Polyethylene**	**6 - 3**	**2 - 12**
Van der Waals	**Waxes**	**0.5 - 1**	**1 - 4**

Importance: deflections, energy absorption, elastic instability

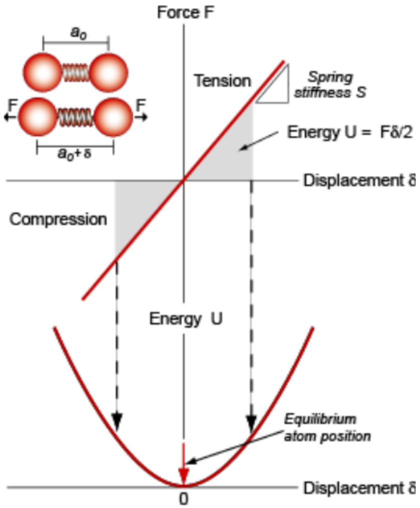


Figure 3. Stretching or compressing an atomic bond raises its energy. Its resistance to stretch is its stiffness, S.

Hooke's law

In one-dimensional linear elasticity, the free energy potential is quadratic with respect to strain

$$\Psi(\varepsilon) = {}^{1}/_{2}\, E\varepsilon^2$$

Therefore, the derived stress becomes linear

$$\sigma(\varepsilon) = \frac{\partial\Psi}{\partial\varepsilon} = \mathrm{E}\varepsilon$$

Where the parameter E is the constant slope or elastic modulus.

This relationship is known as Hooke's law.

Linear Elastic Properties

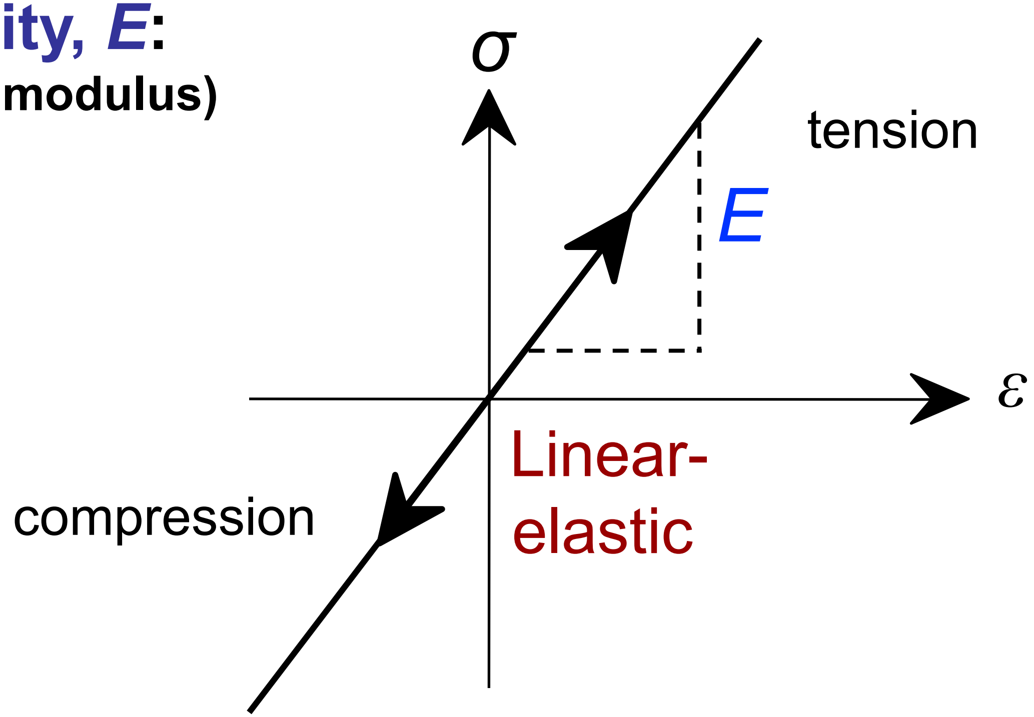
- **Elastic deformation** is nonpermanent and reversible!
  - generally valid at small deformations
  - linear stress strain curve

- **Modulus of Elasticity, *E*:**

  (also known as Young's modulus)

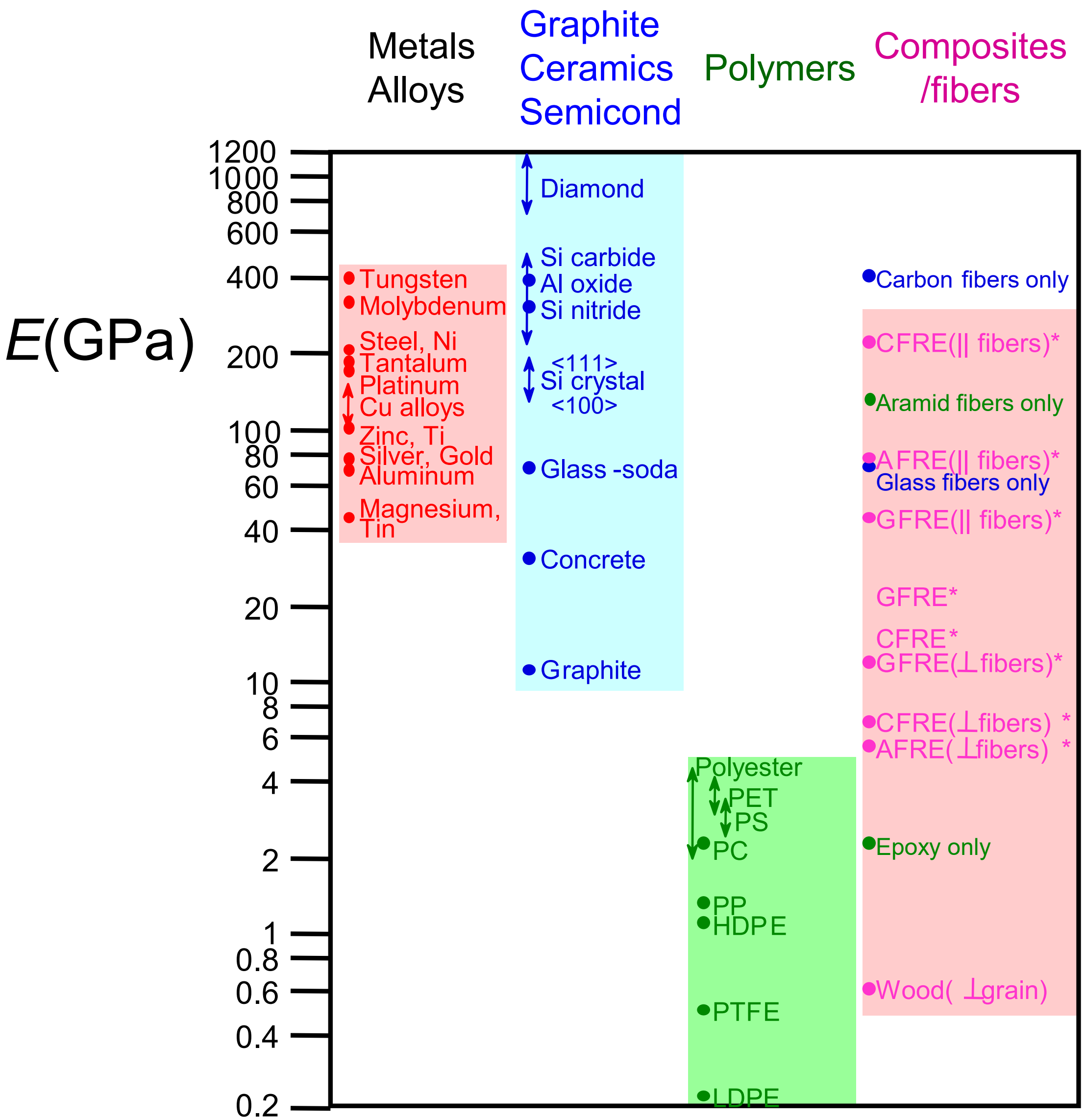
- **Hooke's Law:**

$$\sigma = E\,\varepsilon$$



Units:
E: [GPa] or [psi]
1 GPa = 10^9 Pa

Elastic Modulus – Comparison of Material Types

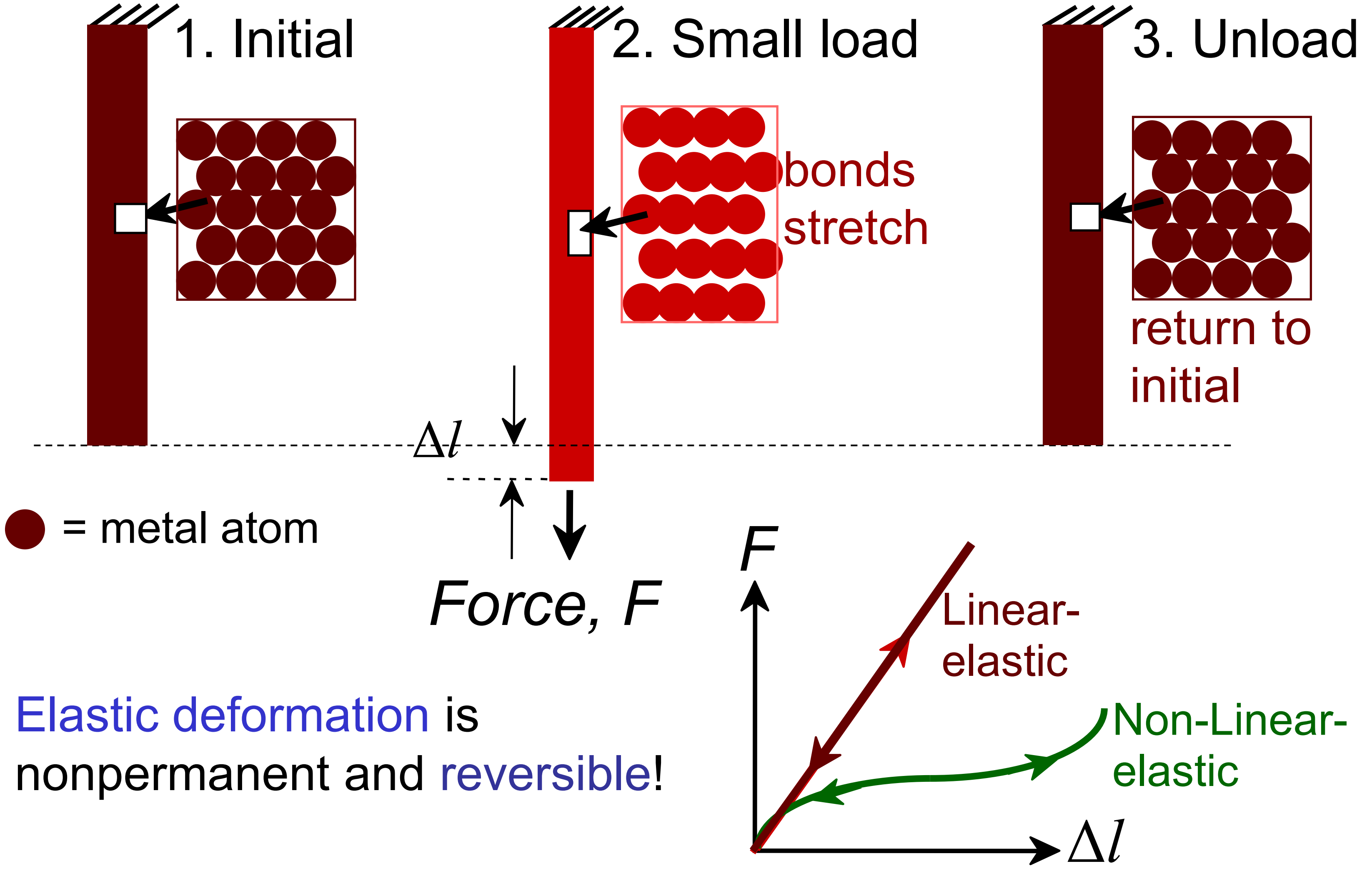


Based on data in Table B.2, *Callister & Rethwisch 10e.*

Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Elastic Deformation

Atomic configurations—before, during, after load (force) application



Elastic deformation is nonpermanent and reversible!

Influence of Bonding Forces

- Elastic modulus depends on interatomic bonding forces
- Modulus proportional to slope of interatomic force-interatomic separation curve $\left(\frac{dF}{dr}\right)_{r_o}$

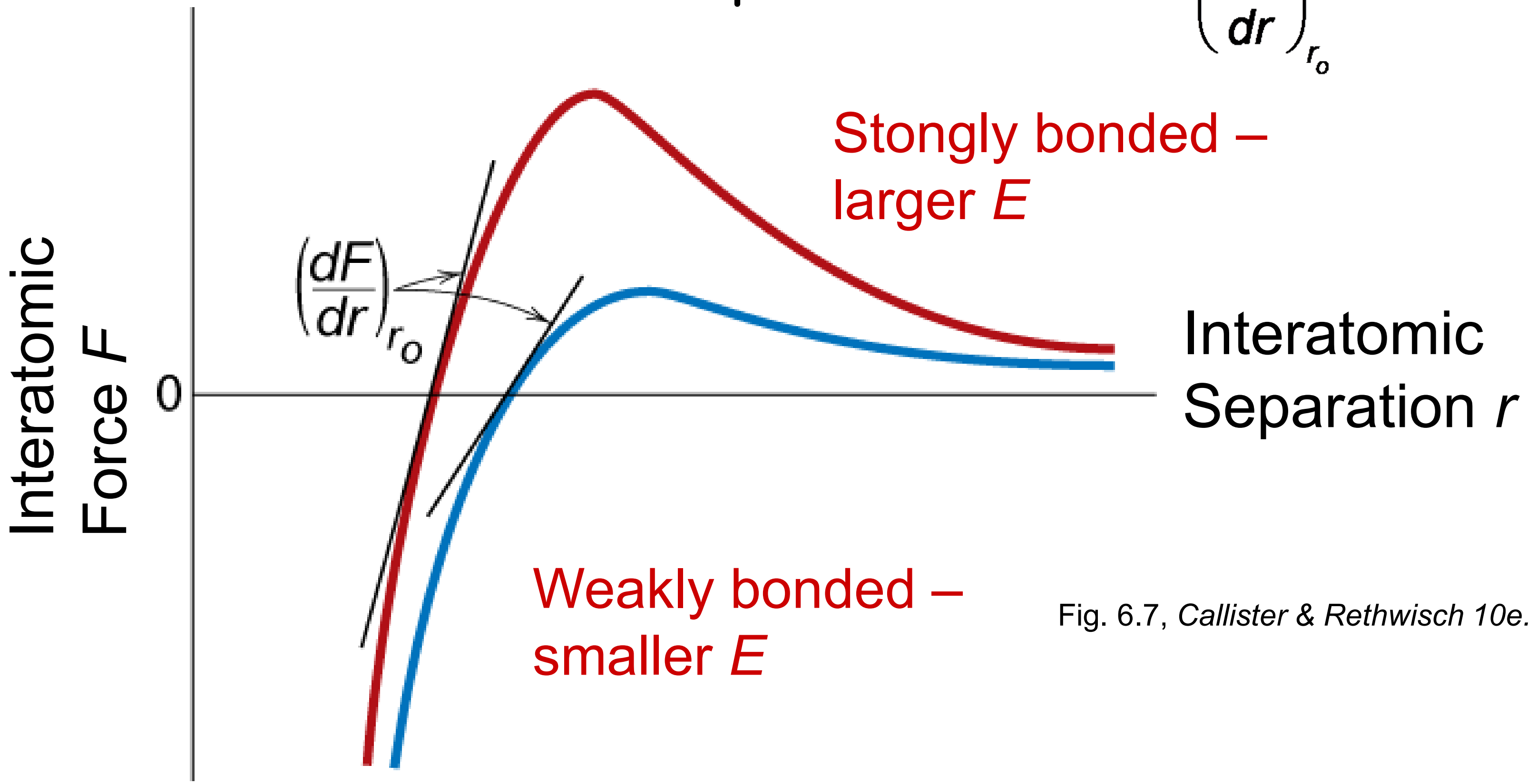


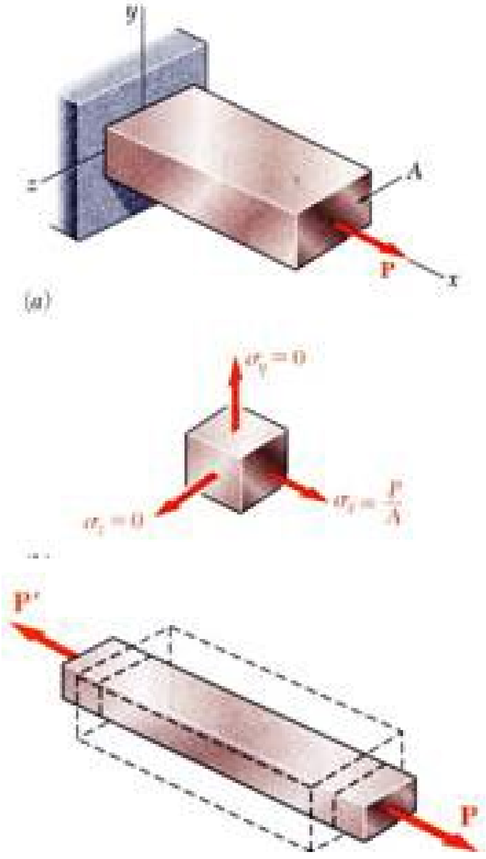
Fig. 6.7, *Callister & Rethwisch 10e.*

Poisson effect

When a uniaxial tensile stress is applied to a solid, it will elongate in the direction of the load.

At the same time, the solid contracts in the direction perpendicular to the applied load.

This phenomenon is known as the Poisson constant effect and is governed by the Poisson constant v.



Poisson's ratio

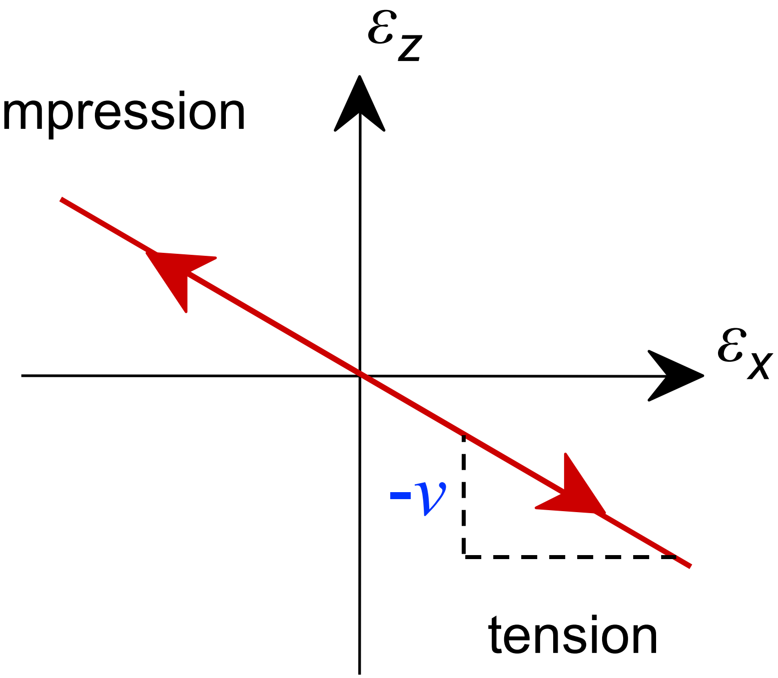
- Poisson's ratio, ν:

$$\nu = -\frac{\varepsilon_z}{\varepsilon_x}$$

metals: $\nu \sim 0.33$

ceramics: $\nu \sim 0.25$

polymers: $\nu \sim 0.40$



Units:
ν: dimensionless

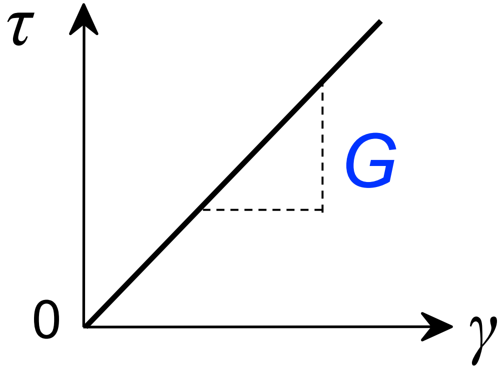
For most metals, ceramics and polymers:

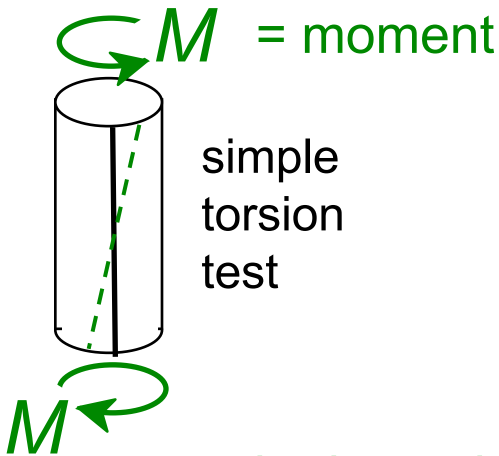
$$0.15 < \nu \leq 0.50$$

Other Elastic Properties

- Elastic Shear modulus, G:

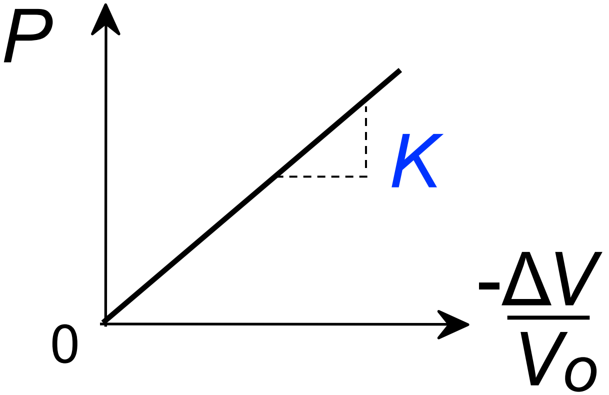
$$\tau = G\gamma$$

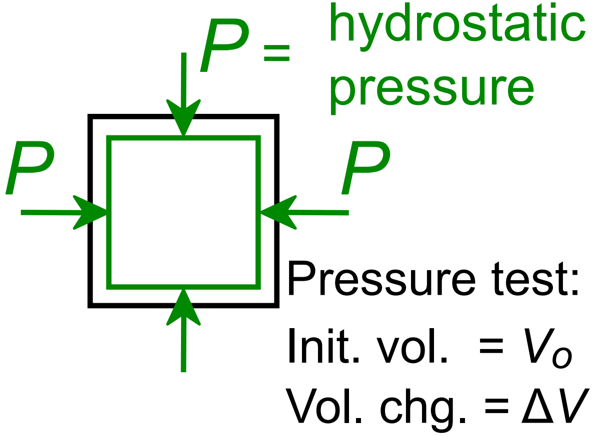




- Elastic Bulk modulus, K:

$$P = -K\frac{\Delta V}{V_o}$$





Pressure test:
Init. vol. = V_o
Vol. chg. = ΔV

- Elastic constant relationships for isotropic materials:

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Elasticity in 3D

In three dimensions, the constant relating the second order stress and strain tensors is a fourth order stiffness tensor:

$$\sigma_{ij}\ (\varepsilon_{ij}) = \mathrm{E}_{ijkl}\varepsilon_{kl}$$

Due to the symmetry of the stress and strain tensors, the (3x3) stress and strain tensors can be projected into (6x1) vectors and the (3x3x3x3) stiffness tensor can be projected into a (6x6) stiffness matrix, which saves significant computation time.

In matrix notation, Hooke's law then takes the following form:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} : \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{bmatrix}$$

Elastic isotropy

For isotropic materials, the elastic response of the material is direction independent. Examples of isotropic materials include amorphous material, but also polycrystals with random orientations.

In this case, Hooke's law simplifies to:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} 2\mu+\lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & 2\mu+\lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & 2\mu+\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{31} \\ 2\varepsilon_{12} \end{bmatrix}$$

with the Lame constant $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$, the Poisson constant ν, and the shear modulus $\mu = \frac{E}{2(1+\nu)}$.

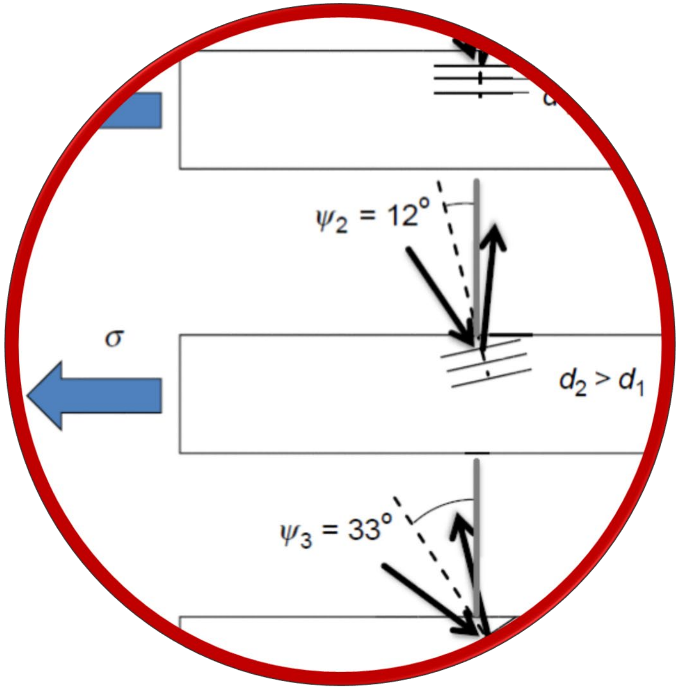
Elastic anisotropy - orthotropy

Many materials, e.g. single crystals, fiber reinforced composites, or polycrystals with a texture feature direction-dependent elastic properties. This phenomenon is called anisotropy. Orthotropy is a special case, where three orthogonal planes of symmetry can be defined. In this case the compliance matrix, the inverse of the stiffness matrix, takes the following form:

$$\mathbb{E} = \begin{pmatrix} \frac{1}{\epsilon_1} & \frac{-\nu_{12}}{\epsilon_1} & \frac{-\nu_{31}}{\epsilon_3} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{\epsilon_1} & \frac{1}{\epsilon_2} & \frac{-\nu_{23}}{\epsilon_2} & 0 & 0 & 0 \\ \frac{-\nu_{31}}{\epsilon_3} & \frac{-\nu_{23}}{\epsilon_2} & \frac{1}{\epsilon_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2\mu_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2\mu_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2\mu_{12}} \end{pmatrix}$$

with 9 orthotropic constants acting as material properties.

Plasticity



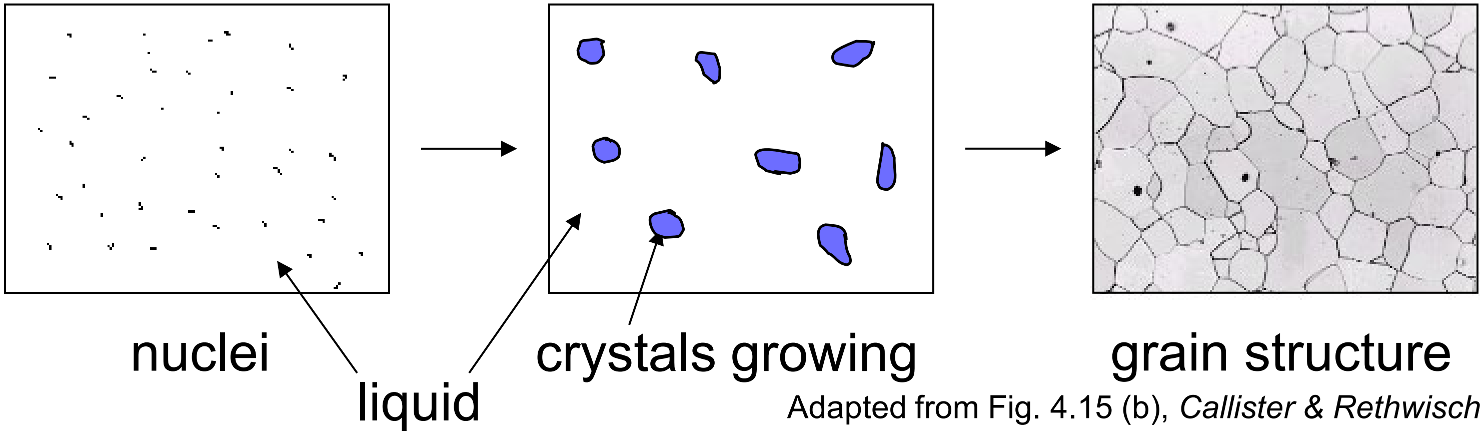
Imperfections in Solids

ISSUES TO ADDRESS...

- What types of defects exist in solid materials?
- How does the number of vacancies depend on temperature?
- What are the two types of solid solutions?
- What are the three types of dislocations?
- What kinds of information come from microscopic examinations?

Solidification

- Solidification- result of casting of molten material
  - 2 steps
    - Nuclei of the solid phase form
    - Crystals grow until their boundaries meet each other – the crystals become grains
- Start with a molten material – all liquid

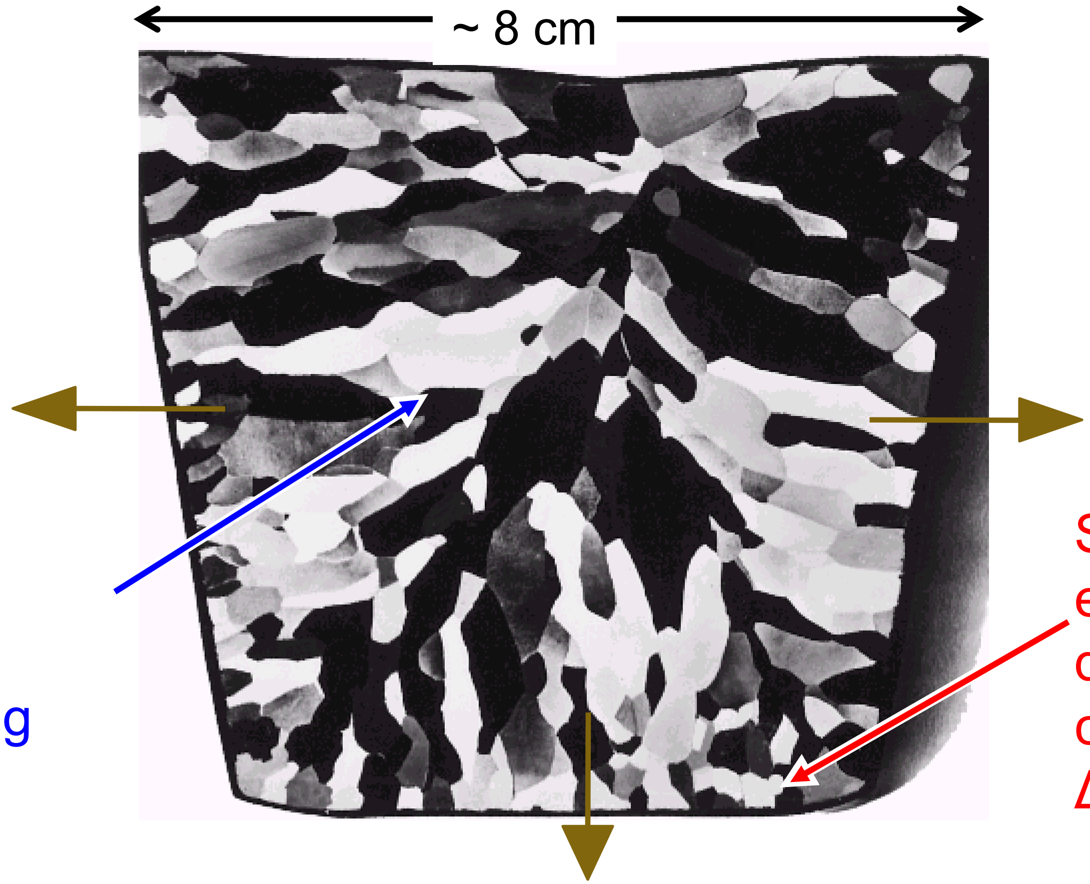


Adapted from Fig. 4.15 (b), *Callister & Rethwisch 10e.*

[Photomicrograph courtesy of L. C. Smith and C. Brady, the National Bureau of Standards, Washington, DC (now the National Institute of Standards and Technology, Gaithersburg, MD.)]

Solidification (continued)

Grains can be - equiaxed (roughly the same dimension in all directions)
- columnar (grains elongated in one direction)



Adapted from Fig. 5.17, *Callister & Rethwisch 3e.* (Reproduced with permission from Metals Handbook, Vol. 9, 9th edition, Metallography and Microstructures, ASM International, Materials Park, OH, 1985.)

heat

flow

Columnar in region with slower cooling

Shell of equiaxed grains due to rapid cooling (greater ΔT) near wall

Grain Refiner - added to make smaller, more uniform, equiaxed grains.

Grains and Grain Boundaries

Grain Boundaries

- regions between grains (crystals)
- crystallographic misalignment across a grain boundary
- Slight atomic disorder
  - high atomic mobility
  - high chemical reactivity

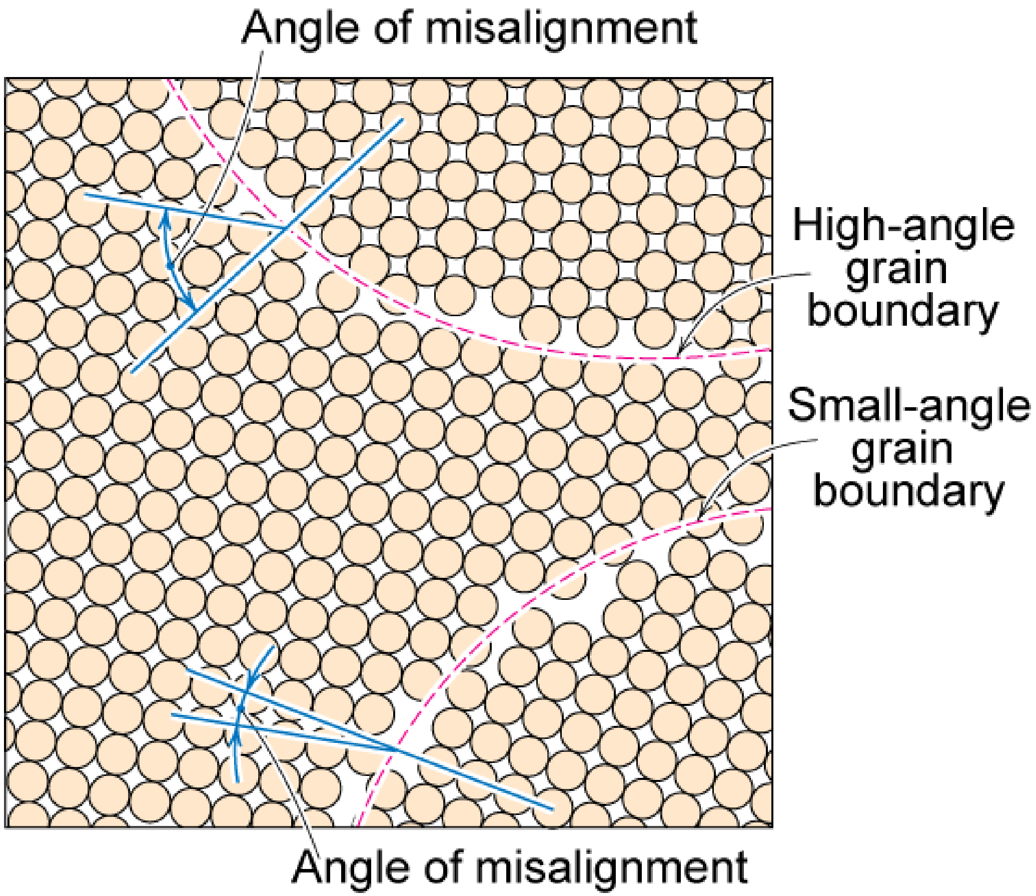


Fig. 4.8, *Callister & Rethwisch 10e.*

Imperfections in Solids

- There is no such thing as a perfect crystal. Crystalline imperfections (or defects) are always present.
- Many of the properties of materials are sensitive to the presence of imperfections.
- Crystalline defect refers to a lattice irregularity with dimensions on the order of an atomic diameter.
- What kinds of crystalline imperfections exist in solids?

Types of Imperfections

- Vacancies
- Interstitial atoms
- Substitutional impurity atoms

Point defects
(0-Dimensional)

- Dislocations

Linear defects
(1-Dimensional)

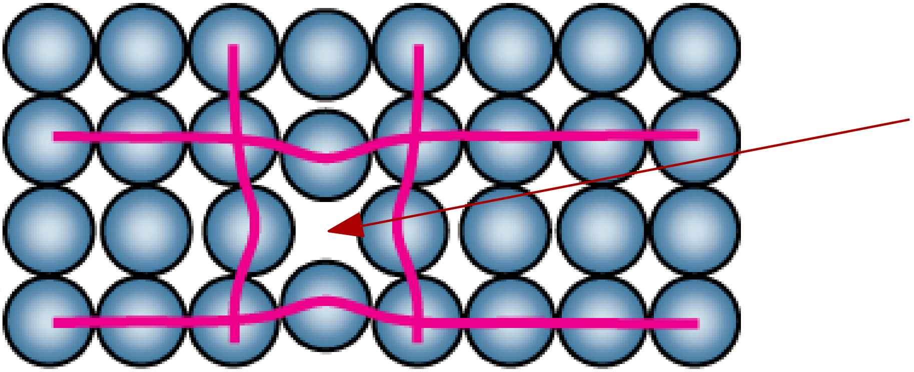
- Grain Boundaries

Interfacial defects
(2-Dimensional)

Point Defects in Metals

- Vacancies:

  -vacant atomic sites.

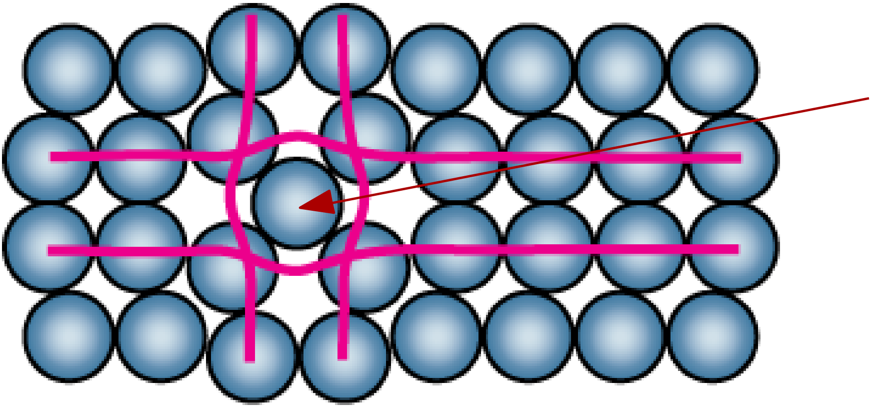


distortion of planes

Vacancy

- Self-Interstitials:

  -Host atoms positioned in interstitial positions between atoms.



distortion of planes

self-interstitial

Vacancies – Computation of Equilibrium Concentration

- Equilibrium concentration varies with temperature!

Number of vacancies

Activation energy

Total number of lattice sites

$$\frac{N_v}{N} = \exp\left(\frac{-Q_v}{kT}\right)$$

Temperature

Boltzmann's constant

(1.38 x 10^{-23} J/atom-K)

(8.62 x 10^{-5} eV/atom-K)

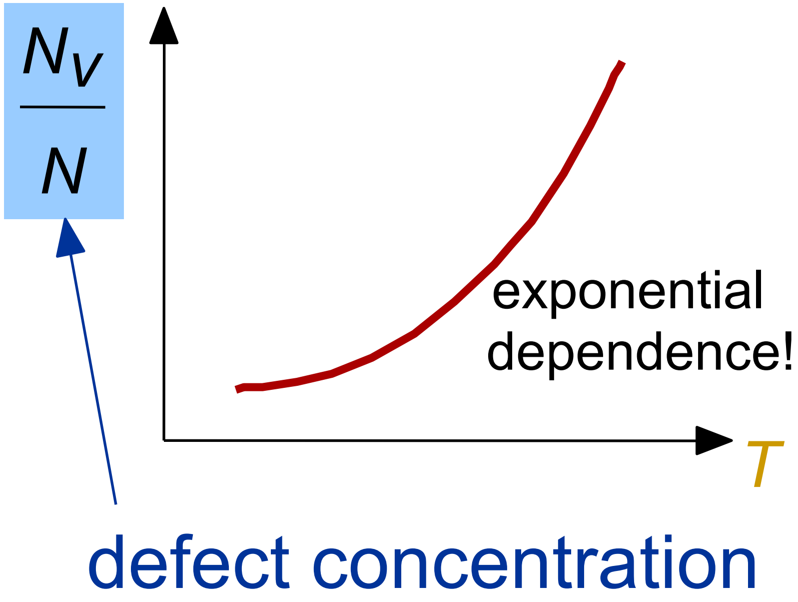
Note: Each lattice site is a potential vacancy.

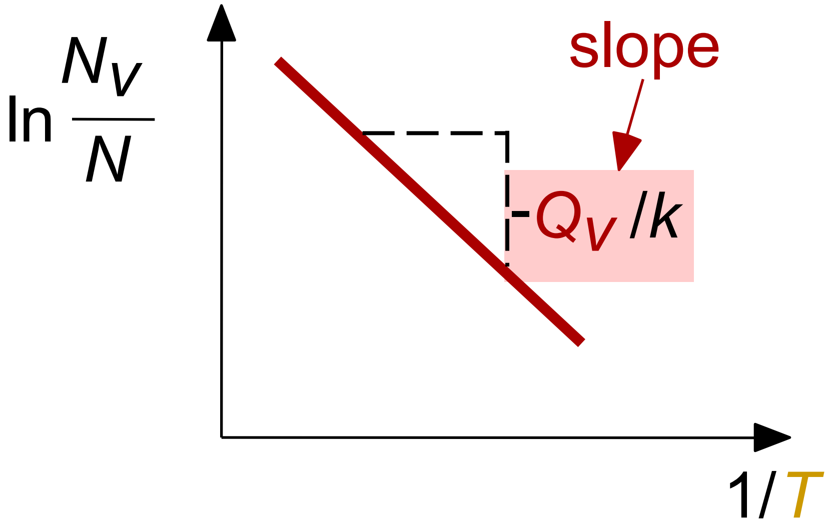
Determination of Activation Energy for Vacancy Formation

- Q_v can be determined experimentally.

$$\frac{N_v}{N} = \exp\left(\frac{-Q_v}{kT}\right)$$

- Data may be plotted as...
- Replot data as follows...





Computation of Equilibrium Vacancy Concentration

- Find the equilibrium number of vacancies in 1 m^3 of Cu at 1000° C.
- Given:

$\rho = 8.4\ g/cm^3$ $A_{Cu} = 63.5\ g/mol$

$Q_V = 0.9\ eV/atom$ $N_A = 6.022 \times 10^{23}\ atoms/mol$

Solution: The first step is to determine the total number of lattice sites N using Equation 4.2

$$N = \frac{N_A \rho}{A_{Cu}} = \frac{(6.022 \times 10^{23}\ \text{sites/mol})(8.4\ \text{g/cm}^3)}{63.5\ \text{g/mol}} \left(\frac{10^6\ \text{cm}^3}{\text{m}^3} \right)$$

$$= 8.0 \times 10^{28}\ \text{sites/m}^3$$

Computation of Equilibrium Vacancy Concentration (continued)

The second step is to determine the equilibrium vacancy concentration N_V using Equation 4.1.

$$N_V = N \exp\left(\frac{-Q_V}{kT}\right) = N \exp\left(\frac{-0.9 \text{ eV/atom}}{(8.62 \times 10^{-5} \text{ eV/atom-K})(1273 \text{ K})}\right)$$

$$= (2.7 \times 10^{-4})\, N$$

- Answer:

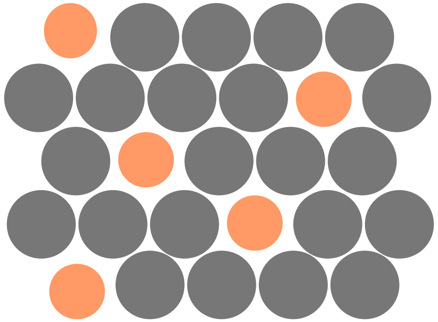
$$N_V = (2.7 \times 10^{-4})(8.0 \times 10^{28}) \text{ sites/m}^3$$

$$= 2.2 \times 10^{25} \text{ vacancies/m}^3$$

Impurities in Metals

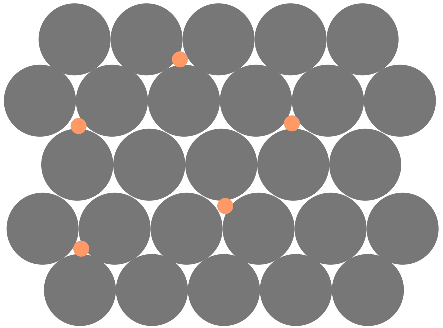
Two outcomes if impurity B atoms are added to a solid composed of host A atoms:

- Solid solution of B in A (i.e., random dist. of B atoms)



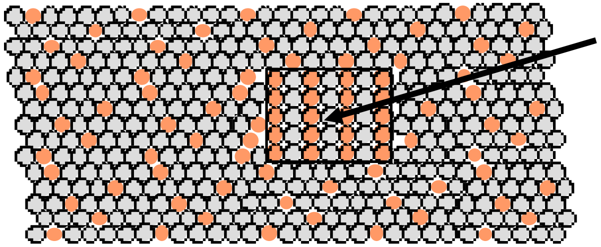
Substitutional solid soln.
(e.g., Cu in Ni)

OR



Interstitial solid soln.
(e.g., C in Fe)

- Solid solution of B in A, plus particles of a new phase (usually for larger concentrations of B)



Second phase particle
-- different composition
-- often different structure.

Impurities in Metals (continued)

Conditions for formation of substitutional solid solutions

W. Hume – Rothery rules

- 1. Δr (atomic radius) < 15%
- 2. Proximity in periodic table
    - i.e., similar electronegativities
- 3. Same crystal structure for pure metals
- 4. Valences
    - *All else being equal, a metal will have a greater tendency to dissolve a metal of higher valence than one of lower valence*

Impurities in Metals (continued)

Application of Hume-Rothery rules – Solid Solutions

Element	*Atomic Radius (nm)*	*Crystal Structure*	*Electro-nega-tivity*	*Valence*
Cu	0.1278	FCC	1.9	+2
C	0.071			
H	0.046			
O	0.060			
Ag	0.1445	FCC	1.9	+1
Al	0.1431	FCC	1.5	+3
Co	0.1253	HCP	1.8	+2
Cr	0.1249	BCC	1.6	+3
Fe	0.1241	BCC	1.8	+2
Ni	0.1246	FCC	1.8	+2
Pd	0.1376	FCC	2.2	+2
Zn	0.1332	HCP	1.6	+2

Table on p. 135, *Callister & Rethwisch 9e.*

Ex: Would you predict more Al or Ag to dissolve in Zn?

1. Δr – slightly favors Al
2. Electronegativity – favors Al
3. Crystal structure – tie
4. Valences –higher valance more soluble so favors Al

This suggests Al is more soluble in Zn. This agrees with experimental observations.

Specification of Composition

- weight percent

$$C_1 = \frac{m_1}{m_1 + m_2} \text{ x } 100$$

m_1 = mass of component 1

- atom percent

$$C_1^{'} = \frac{n_{m1}}{n_{m1} + n_{m2}} \text{ x } 100$$

n_{m1} = number of moles of component 1

Linear Defects—Dislocations

Dislocations

- Are one-dimensional defects around which atoms are misaligned

- Edge dislocation:
  - extra half-plane of atoms inserted in a crystal structure
  - **b** perpendicular (⊥) to dislocation line
- Screw dislocation:
  - spiral planar ramp resulting from shear deformation
  - **b** parallel (||) to dislocation line

Burger’s vector, **b**: measure of lattice distortion

Edge Dislocation

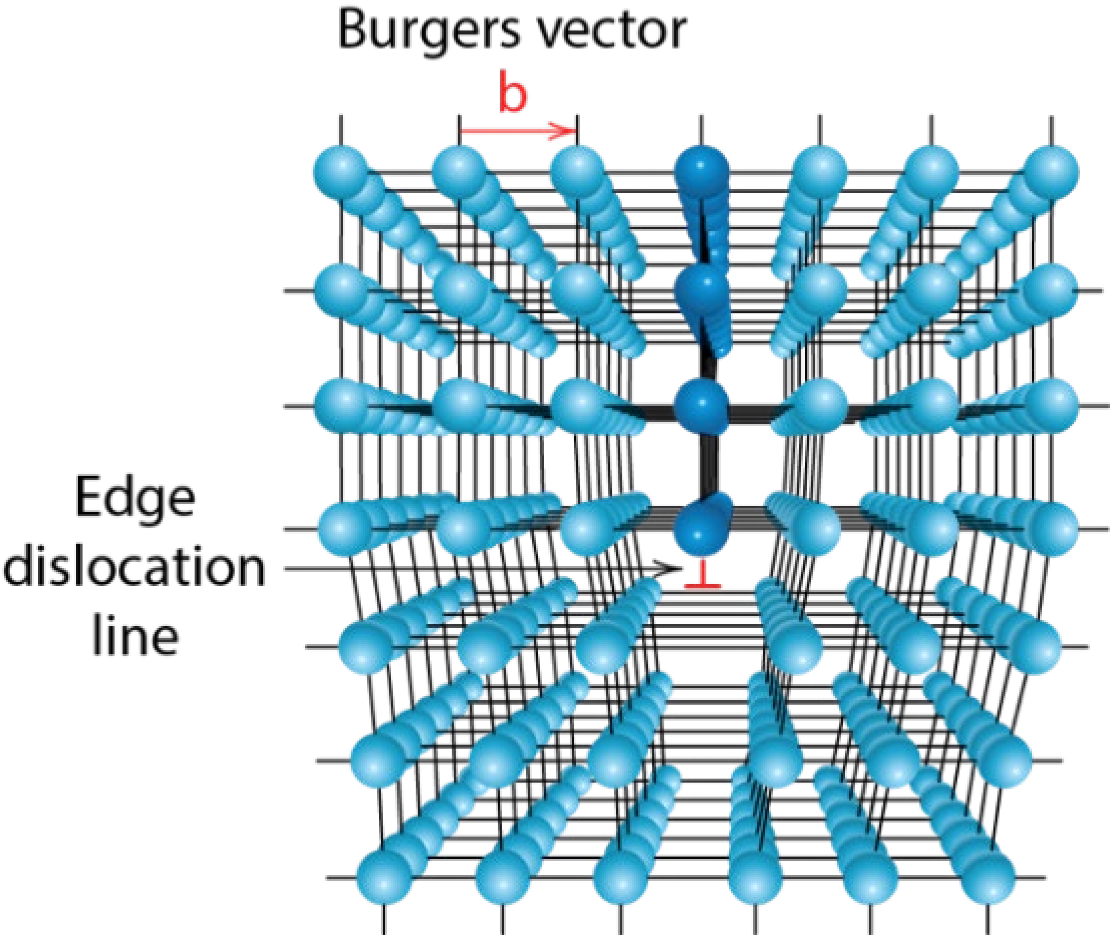
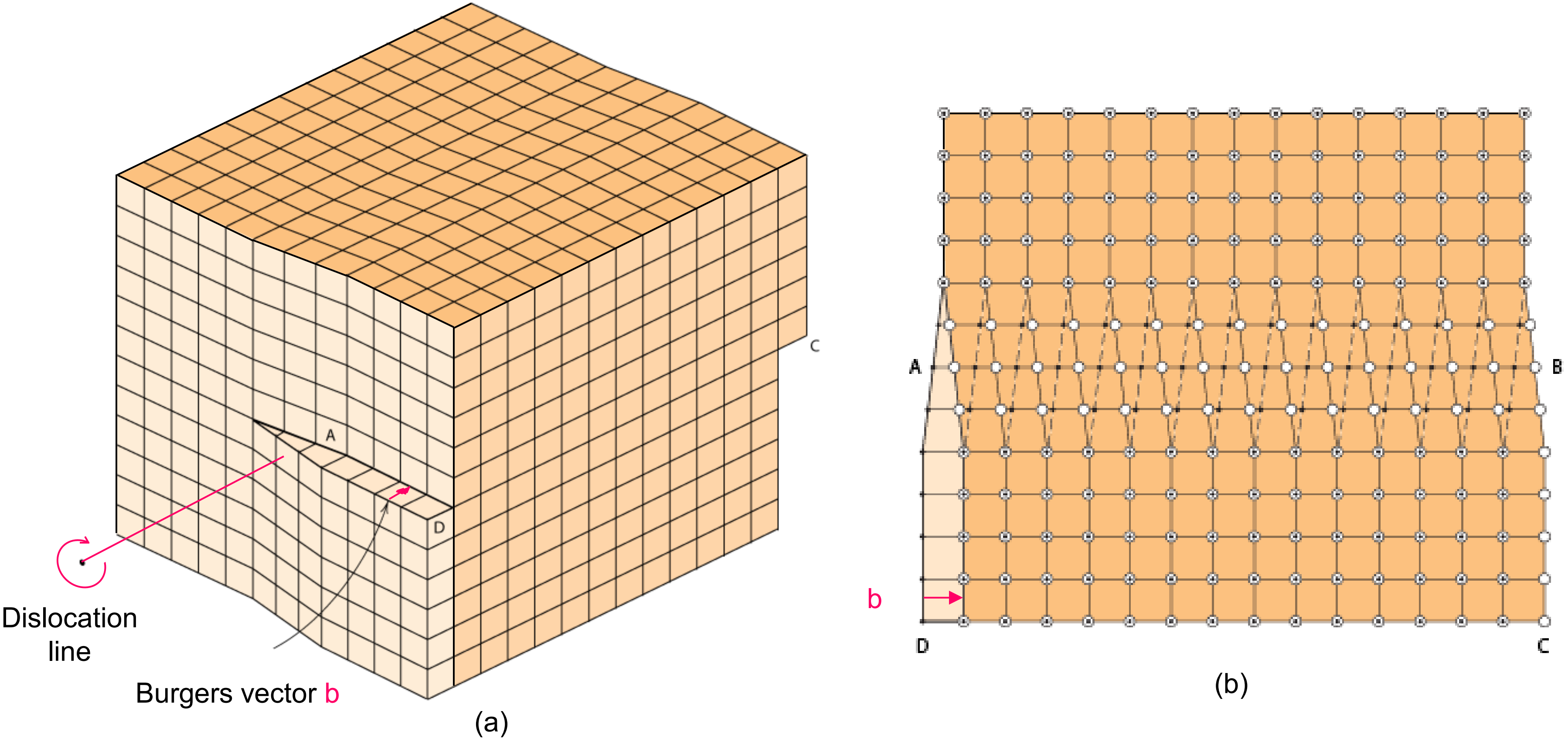


Fig. 4.4, *Callister & Rethwisch 10e.*

Screw Dislocation

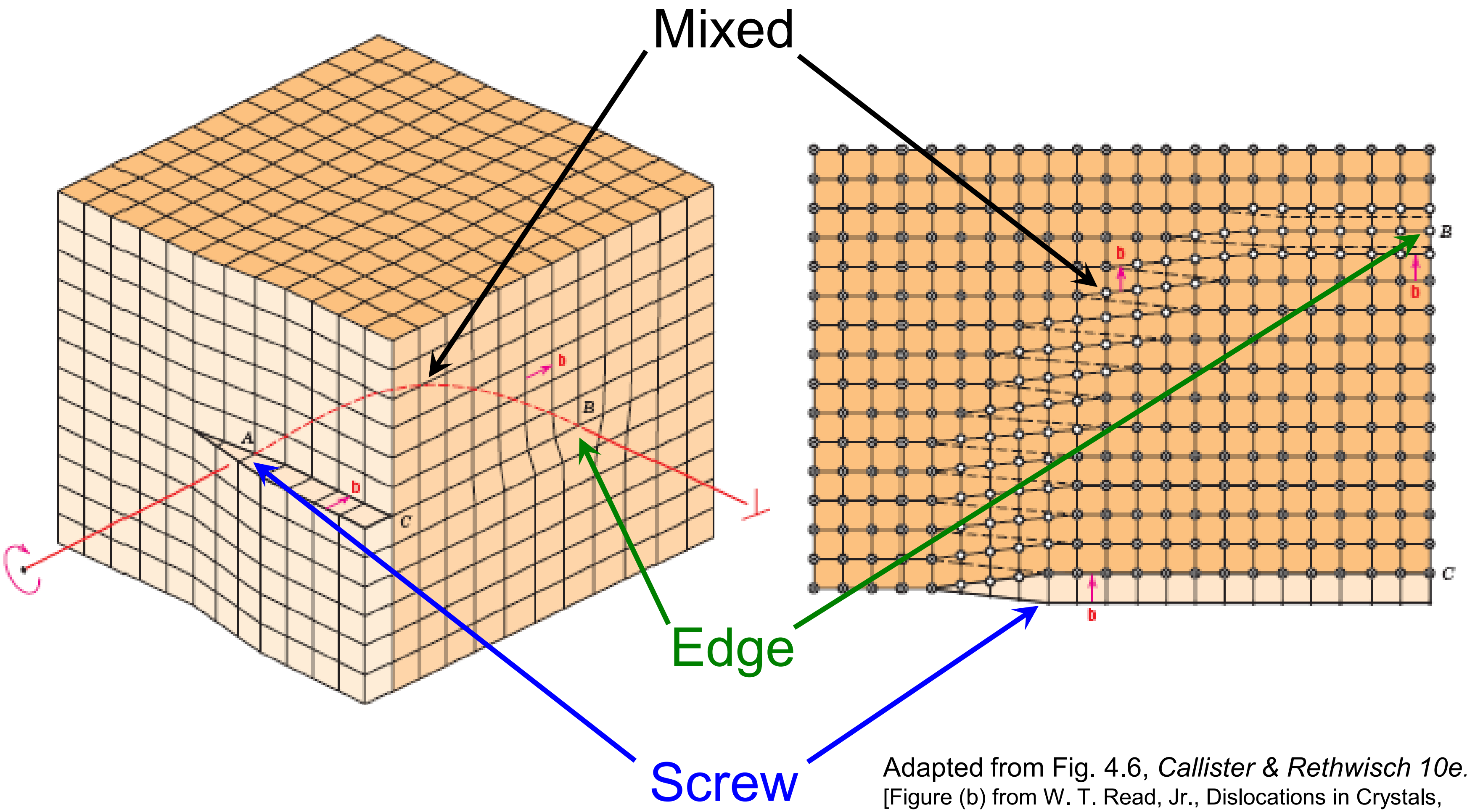
(a) Schematic of screw dislocation in a crystal

(b) Top view of screw dislocation in (a)



Adapted from Fig. 4.5, *Callister & Rethwisch 10e.*
[Figure (b) from W. T. Read, Jr.,Dislocations in Crystals, McGraw-Hill Book Company, New York, NY, 1953.]

Edge, Screw, and Mixed Dislocations



Mixed

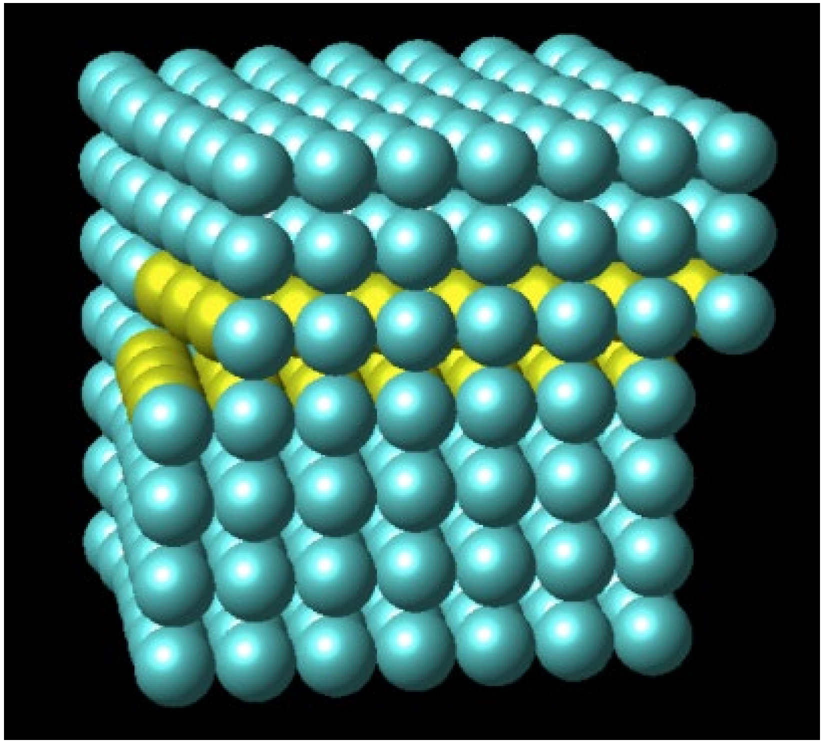
Edge

Screw

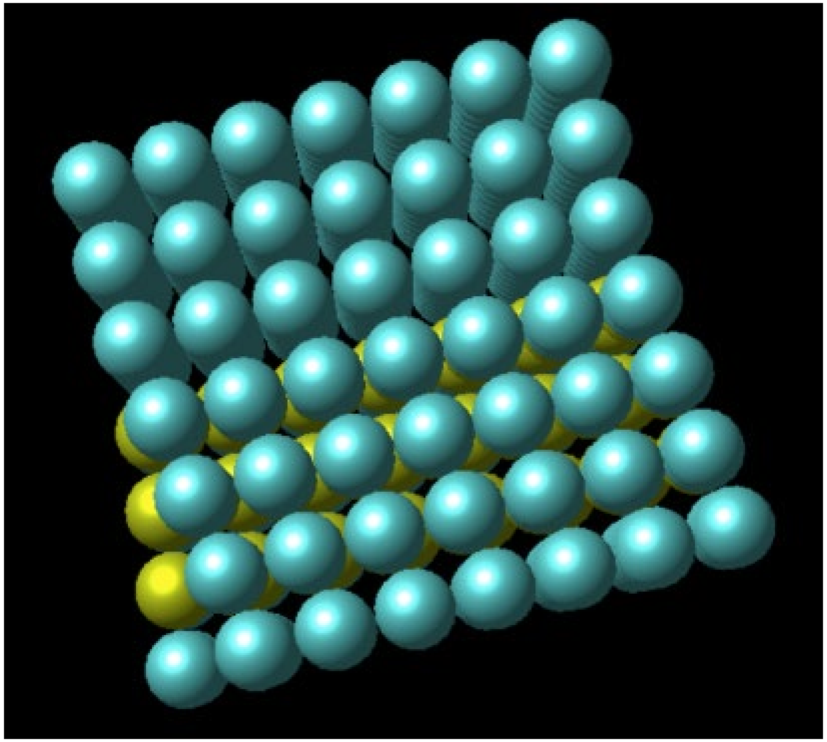
Adapted from Fig. 4.6, *Callister & Rethwisch 10e.*
[Figure (b) from W. T. Read, Jr., Dislocations in Crystals, McGraw-Hill Book Company, New York, NY, 1953.]

VMSE Screenshots of a Screw Dislocation

- In VMSE:
  - crystal region containing screw dislocation—rotated by clicking-and-dragging
  - dislocation motion may be animated



Front View



Top View

Observation of Dislocations

Dislocations appear as dark lines in this electron micrograph

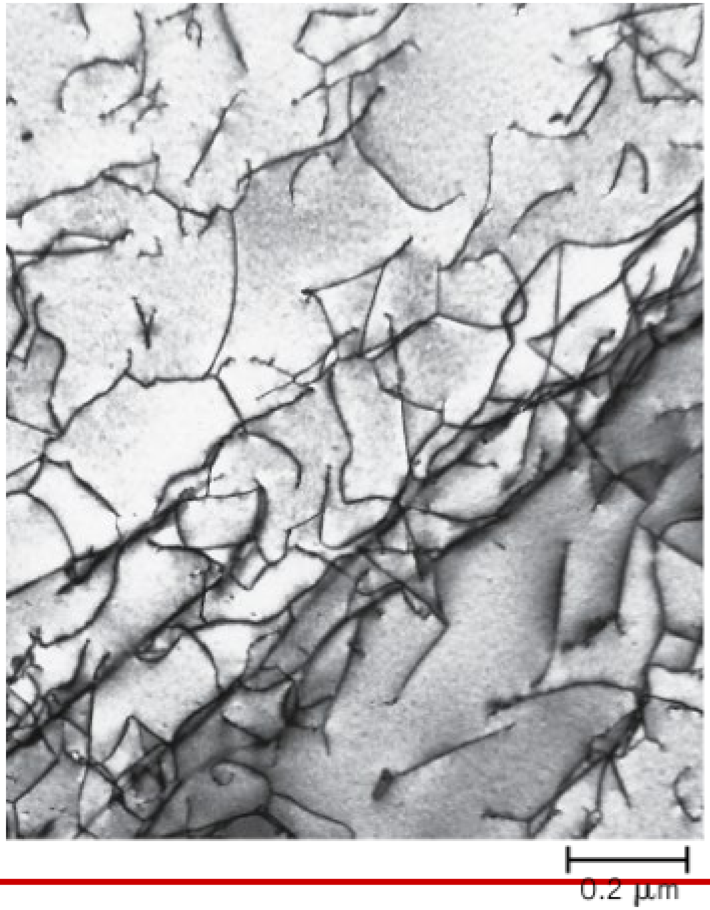


Fig. 4.7, *Callister & Rethwisch 10e.*
(Courtesy of M. R. Plichta, Michigan Technological University.)

Linear Defects—Dislocations

Dislocations:

- move when stresses are applied,
- permanent (plastic) deformation results from dislocation motion.

Schematic of a single crystal metal

- unstressed (undeformed)

- after tensile elongation (after plastic deformation)

Steps correspond to plastic deformation: each step is produced by dislocations that have moved to the crystal surface.

Interfacial (Planar) Defects

- Twin boundaries (or planes)
  - Mirror reflections of atom positions of one side of twin plane to the other side.

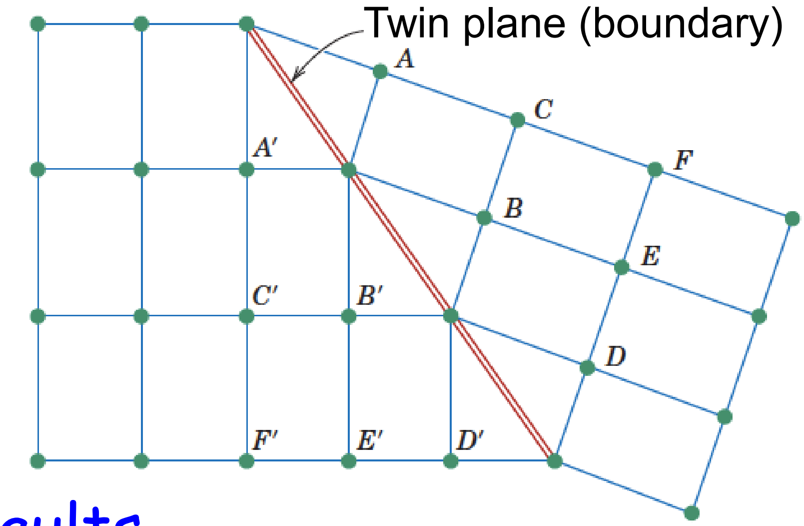


Fig. 4.10, *Callister & Rethwisch 9e.*

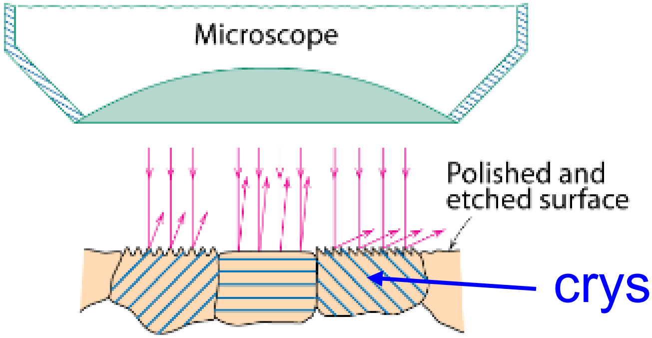
- Stacking faults
  - Occur when there is an error in the planar stacking sequence
  - Ex: for FCC metals
    - normal sequence is ABCABC
    - becomes ABCABABC when there is a packing fault

Microscopic Examination

- Grain size is an important microscopic characteristic.
- Grain size can vary from one material to another.
  - Grain sizes can be quite large
    - ex: large single crystal of quartz or diamond or Si; individual grains visible in aluminum light posts and garbage cans
  - Grain sizes can be quite small (< mm); necessary to observe with a microscope.

Optical Microscopy

- Uses light – useful up to 2000X magnification.
- Polishing removes surface features (e.g., scratches)
- Etching changes reflectance, depending on grain orientation.



crystallographic planes

Fig. 4.14(b) & (c), *Callister & Rethwisch 10e.*

Courtesy of J.E. Burke, General Electric Co.

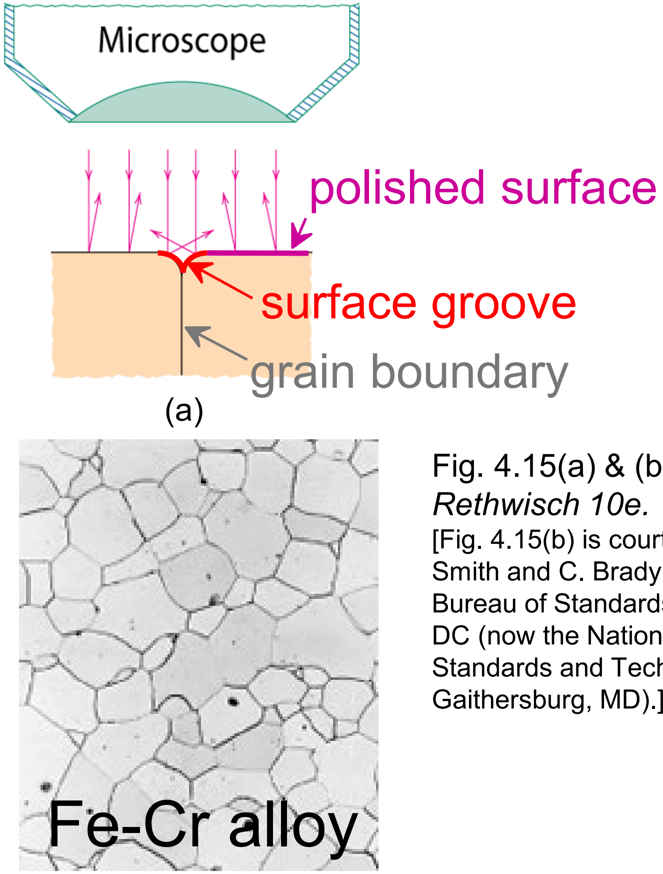
Microstructure of a brass alloy (a Cu-Zn alloy)

grain

Optical Microscopy (cont.)

Grain boundaries...

- are more susceptible to etching
- after etching, grain boundaries appear as dark lines



(a)

(b)

Fig. 4.15(a) & (b), *Callister & Rethwisch 10e.*
[Fig. 4.15(b) is courtesy of L.C. Smith and C. Brady, the National Bureau of Standards, Washington, DC (now the National Institute of Standards and Technology, Gaithersburg, MD).]

Optical Microscopy

- Polarized light
  - metallographic scopes often use polarized light to increase contrast
  - Also used for transparent samples such as polymers

Electron Microscopy

- Best resolution for optical microscopes is ≈ 0.1 µm (100 nm)
- For higher resolution need to use shorter wavelength radiation
  - X-Rays? Difficult to focus.
  - Electron beams
    - Wavelengths as short as 3 pm (0.003 nm) possible
      - (Magnification as high as 1,000,000X are achievable)
    - Atomic resolution possible
    - Electron beams focused by magnetic lenses.

Summary

- Point, Linear, and Interfacial defects exist in solids.
  - Point defects
    - Vacancies
    - Interstitial atoms
    - Substitutional impurity atoms
  - Linear defects
    - Dislocations
  - Interfacial defects
    - Grain boundaries
    - Twin boundaries
    - Stacking Faults
- The equilibrium number vacancy defects depends on temperature

$$N_v = N \exp\left(-\frac{Q_v}{kT}\right)$$

- Dislocation types include edge, screw, and mixed

Mechanical Properties of Metals

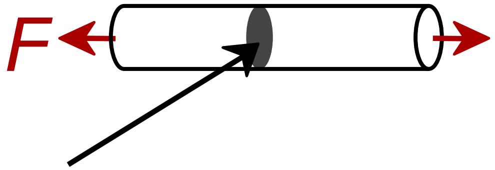
ISSUES TO ADDRESS...

- When a metal is exposed to mechanical forces, what parameters are used to express force magnitude and degree of deformation?
- What is the distinction between elastic and plastic deformations?
- How are the following mechanical characteristics of metals measured?
  - (a) Stiffness
  - (b) Strength
  - (c) Ductility
  - (d) Hardness
- What parameters are used to quantify these properties?

Common States of Stress

- **Simple tension: cable**

Ski lift (photo courtesy P.M. Anderson)



F = force

A_o = cross-sectional area of cable (with no load)

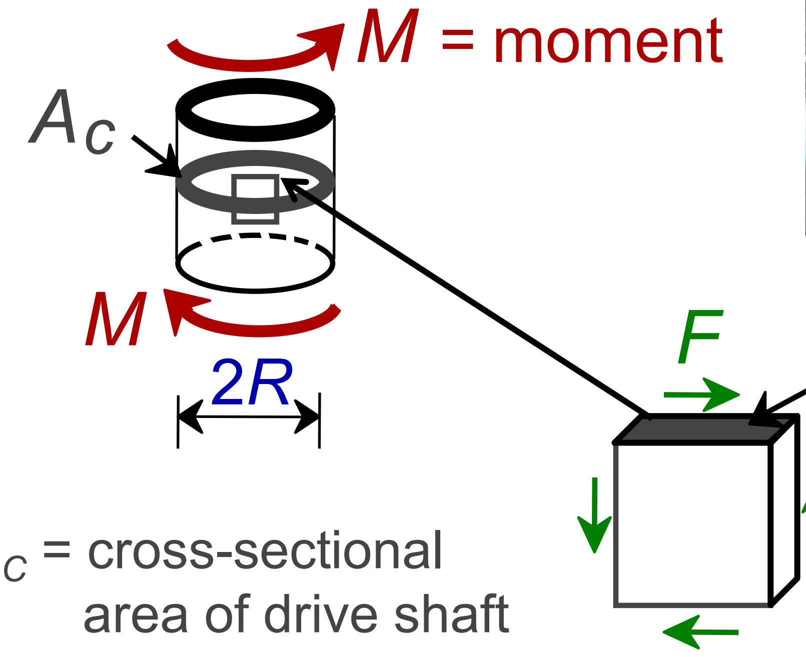
Tensile stress = σ

$$\sigma = \frac{F}{A_0}$$

Common States of Stress (cont.)

- **Torsion (a form of shear):**
  drive shaft

Ski lift (photo courtesy P.M. Anderson)

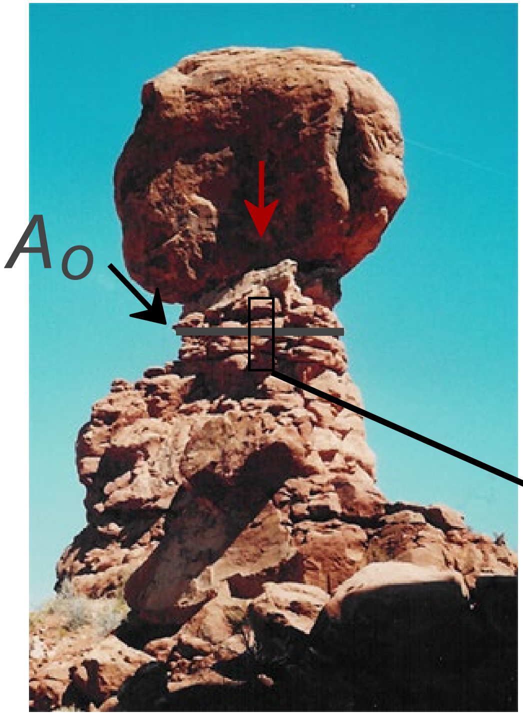


A_C = cross-sectional area of drive shaft (with no load)

$$\tau = \frac{F}{A_s} = \frac{M}{A_c R}$$

OTHER COMMON STRESS STATES (i)

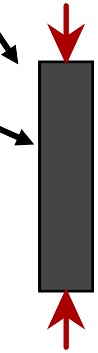
- **Simple compression:**



Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)

Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

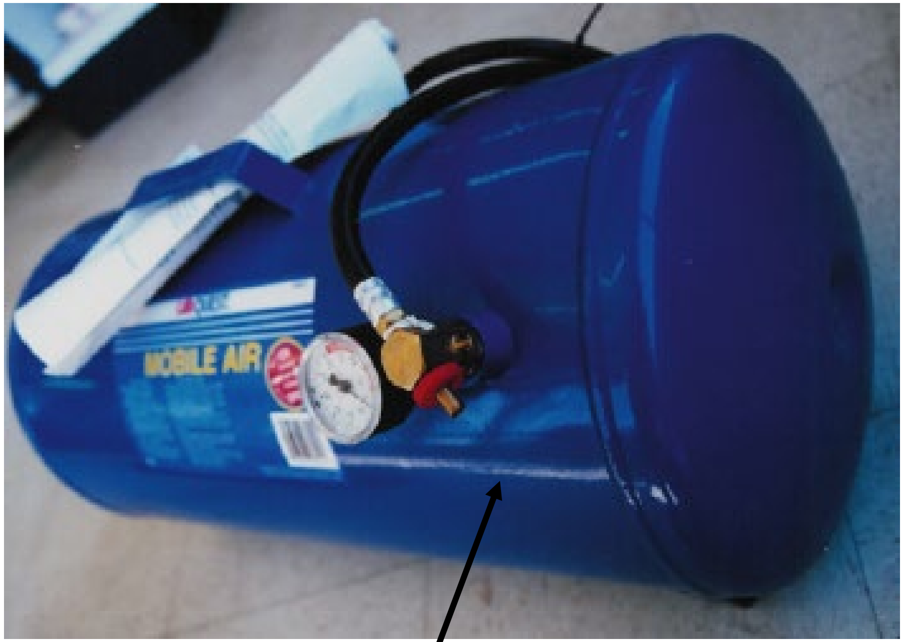
$$\sigma = \frac{F}{A_o}$$



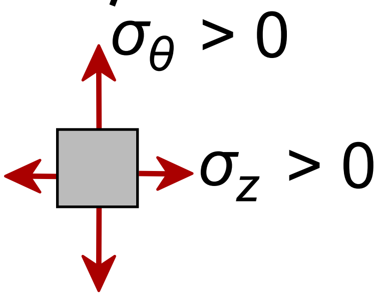
Note: structure members are under compression ($F < 0$ and $\sigma < 0$).

OTHER COMMON STRESS STATES (ii)

- **Bi-axial tension:**



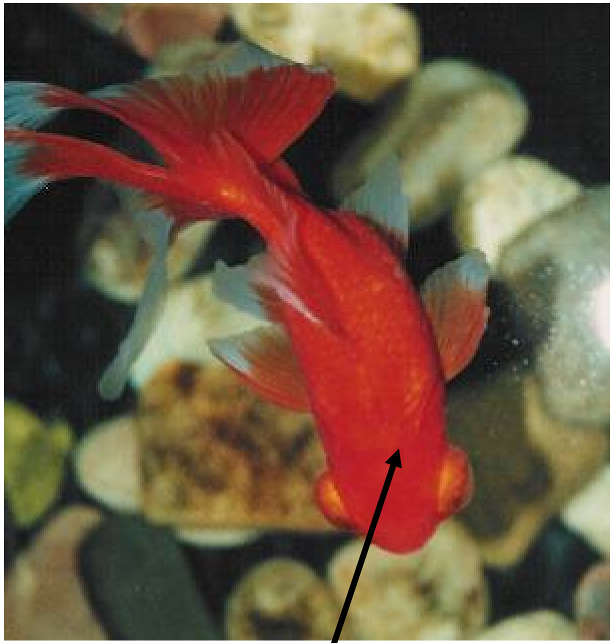
Pressurized tank
(photo courtesy
P.M. Anderson)



$\sigma_\theta > 0$

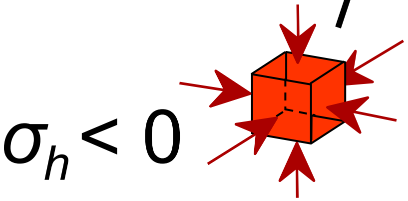
$\sigma_z > 0$

- **Hydrostatic compression:**



Fish under water

(photo courtesy
P.M. Anderson)



$\sigma_h < 0$

Stress-Strain Testing

- **Typical tensile test machine**

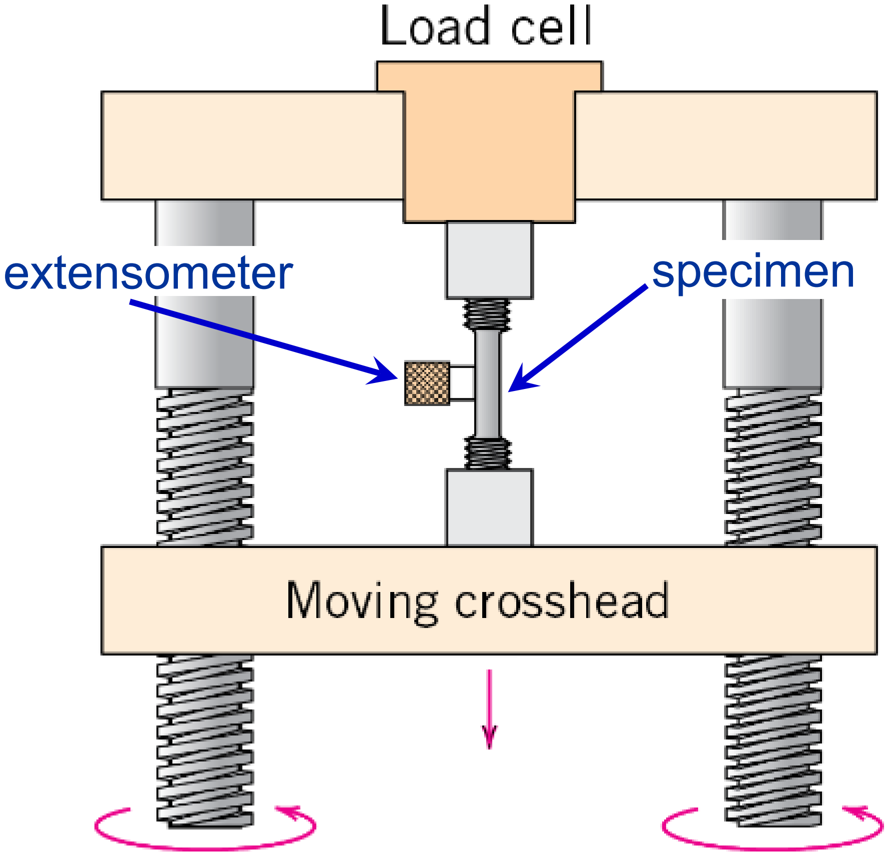


Fig. 6.3, *Callister & Rethwisch 10e.*

(Taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

- **Typical tensile specimen**

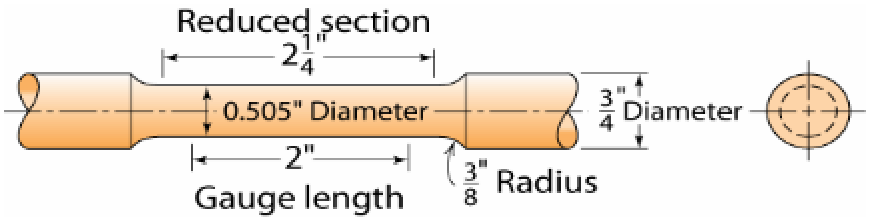
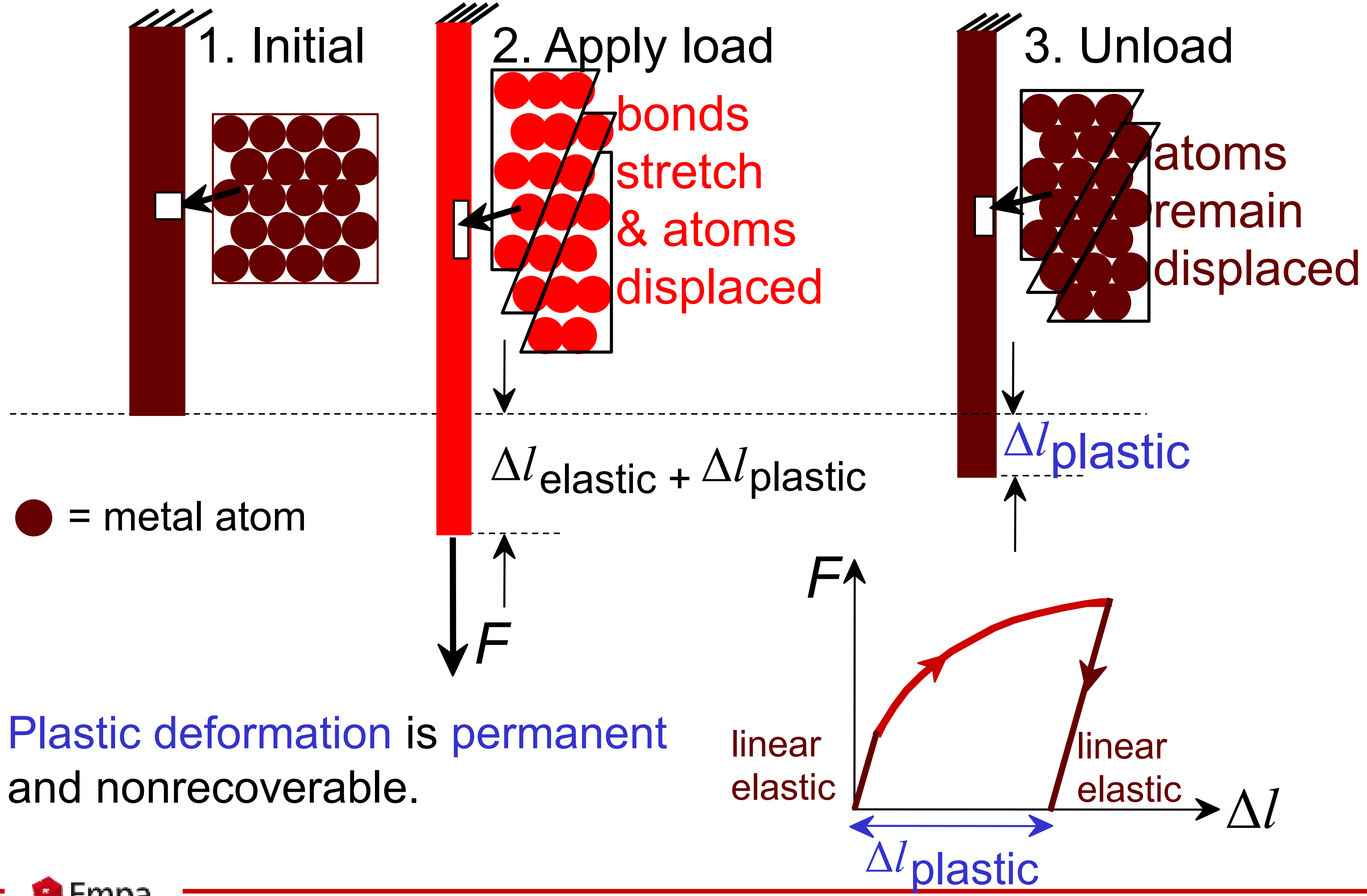


Fig. 6.2, *Callister & Rethwisch 10e.*

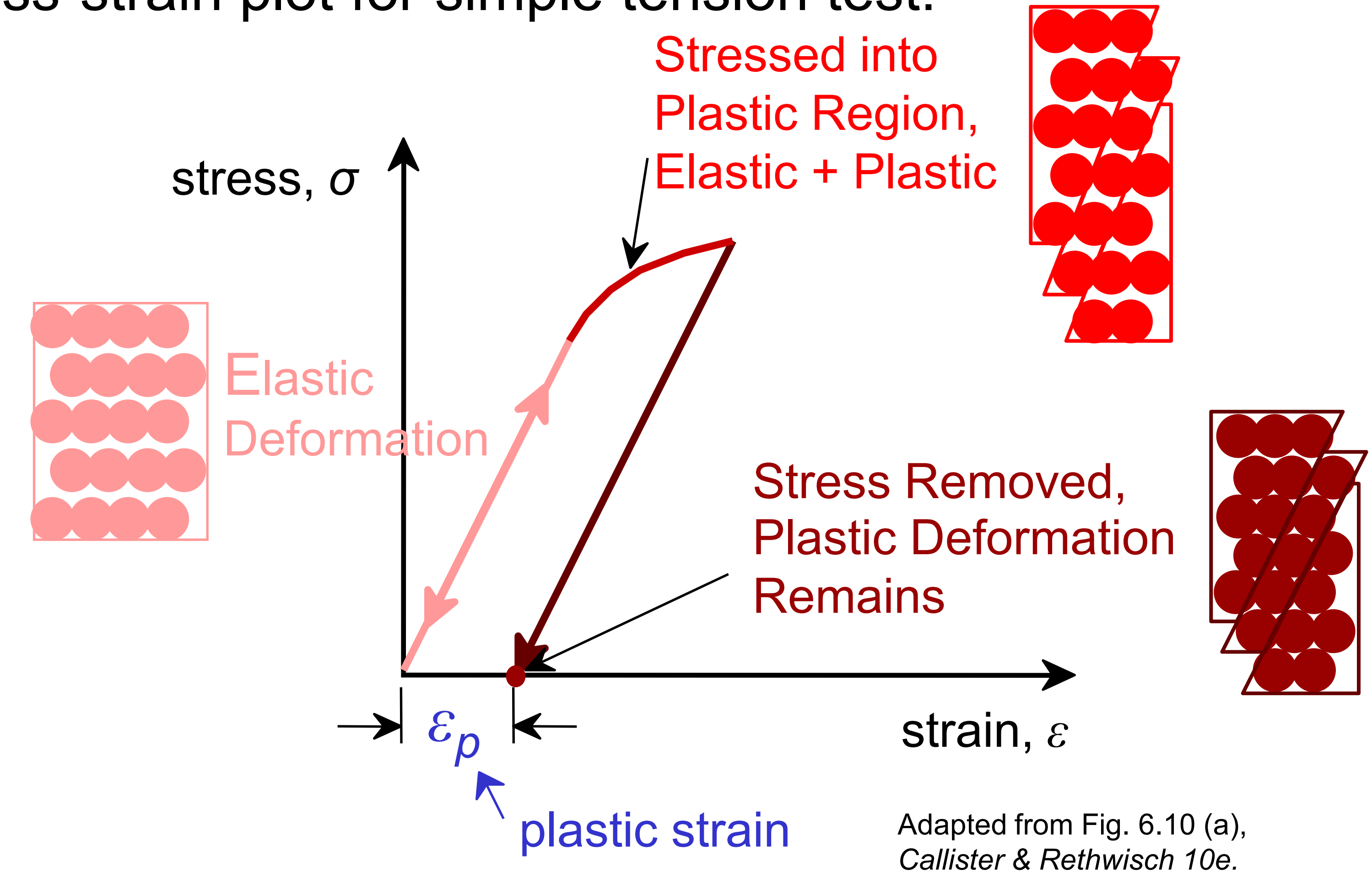
Plastic Deformation (Metals)



Plastic deformation is permanent and nonrecoverable.

Plastic Deformation

- Plastic Deformation is permanent and nonrecoverable
- Stress-strain plot for simple tension test:

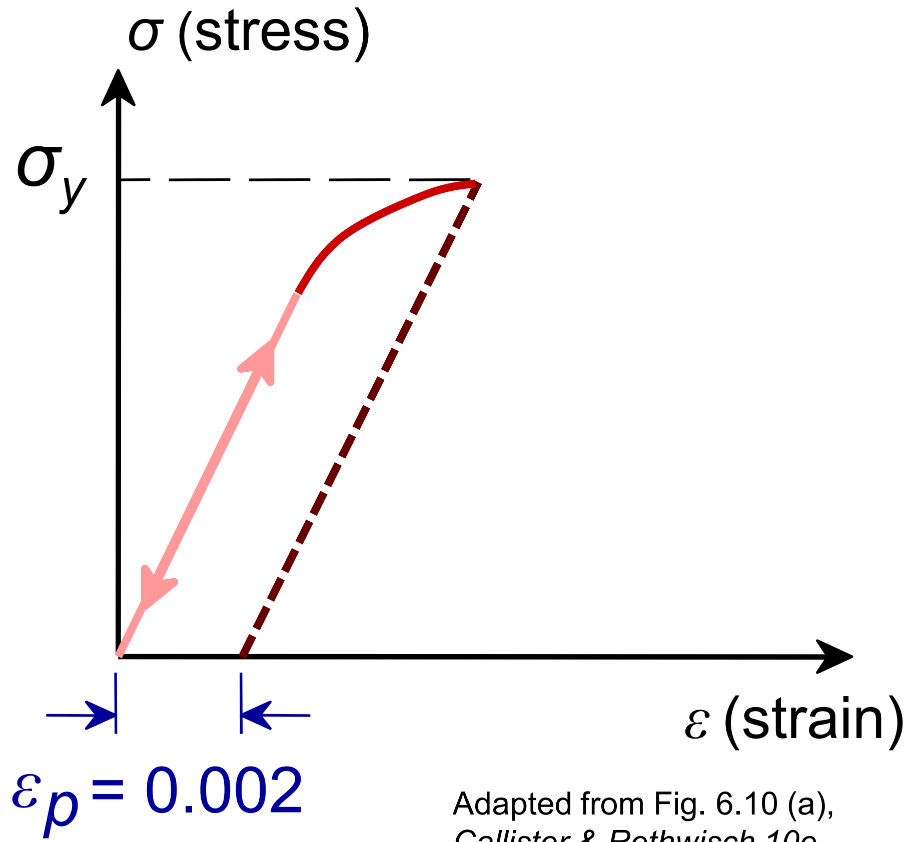


Adapted from Fig. 6.10 (a),
Callister & Rethwisch 10e.

Yield Strength

- Transition from elastic to plastic deformation is gradual
- Yield strength = stress at which *noticeable* plastic deformation has occurred

when $\varepsilon_p = 0.002$



σ_y = yield strength

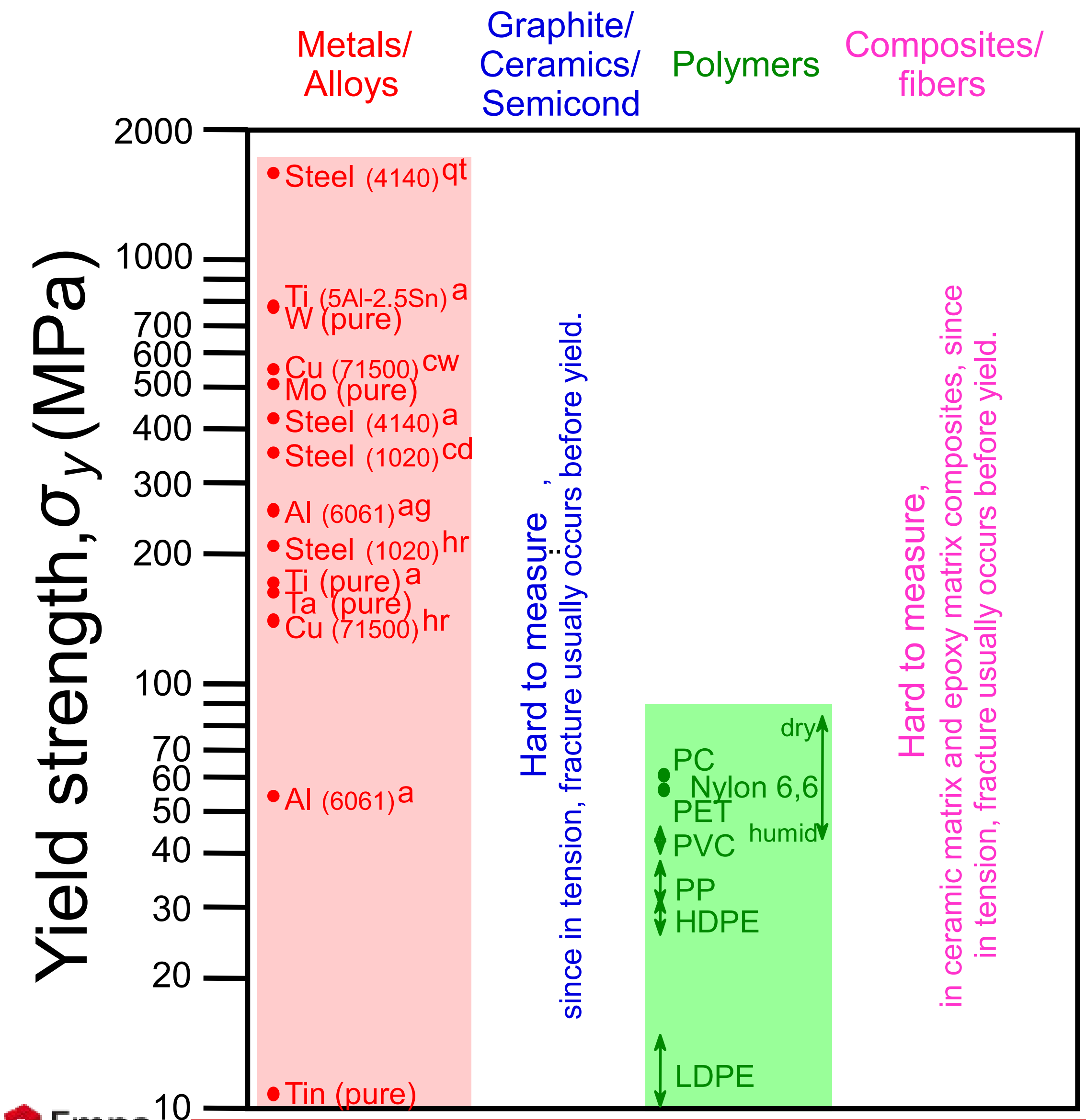
Note: for 5 cm sample

$\varepsilon = 0.002 = \Delta z/z$

$\Delta z = 0.01$ cm

Adapted from Fig. 6.10 (a), *Callister & Rethwisch 10e.*

Yield Strength – Comparison of Material Types



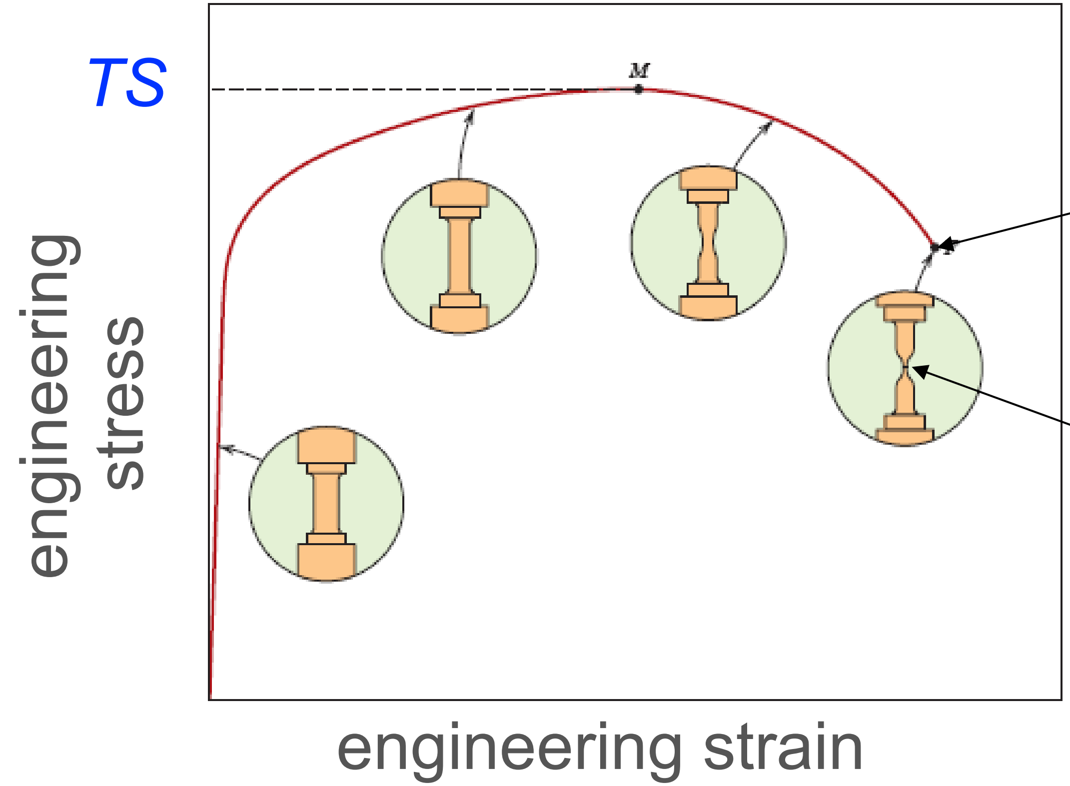
Room temperature values

Based on data in Table B.4, *Callister & Rethwisch 10e.*

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

Tensile Strength

- Tensile strength (*TS*) = maximum stress on engineering stress-strain curve.



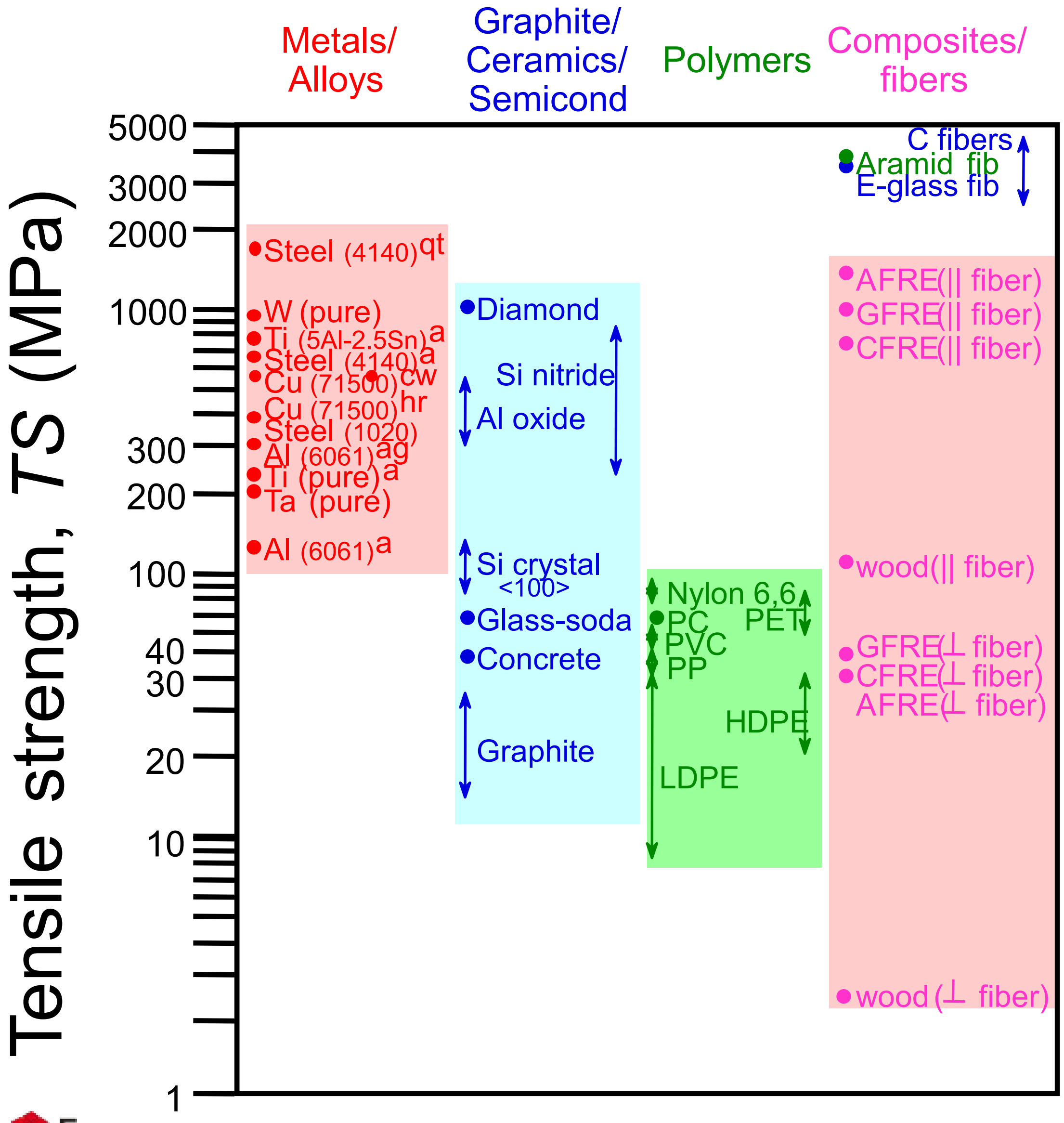
Adapted from Fig. 6.11, *Callister & Rethwisch 10e.*

Fracture strength

Neck – acts as stress concentrator

- Metals: Maximum on stress-strain curve appears at the onset of noticeable necking

Tensile Strength: Comparison of Material Types



Room temperature values

Based on data in Table B4, *Callister & Rethwisch 10e.*

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

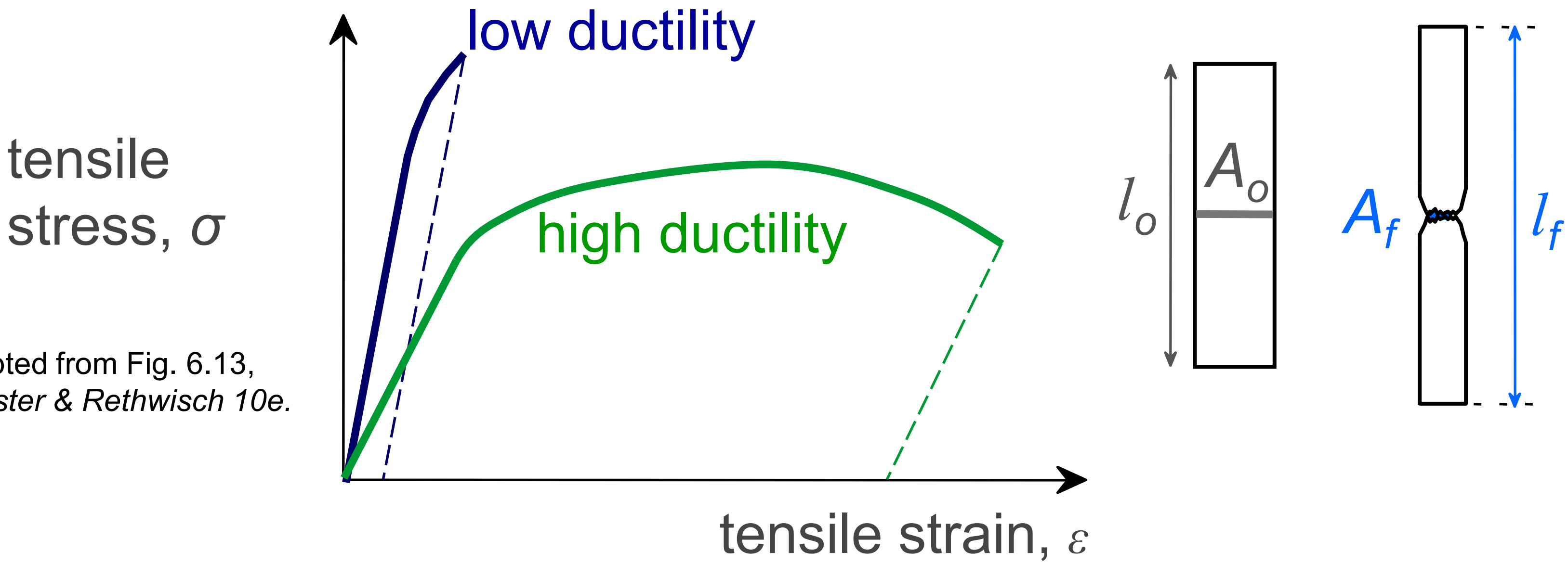
AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

Ductility

- Ductility = amount of plastic deformation at failure:
- Specification of ductility
  - -- Percent elongation:
  - -- Percent reduction in area:

$$\%EL = \frac{l_f - l_0}{l_0} \text{ x } 100$$

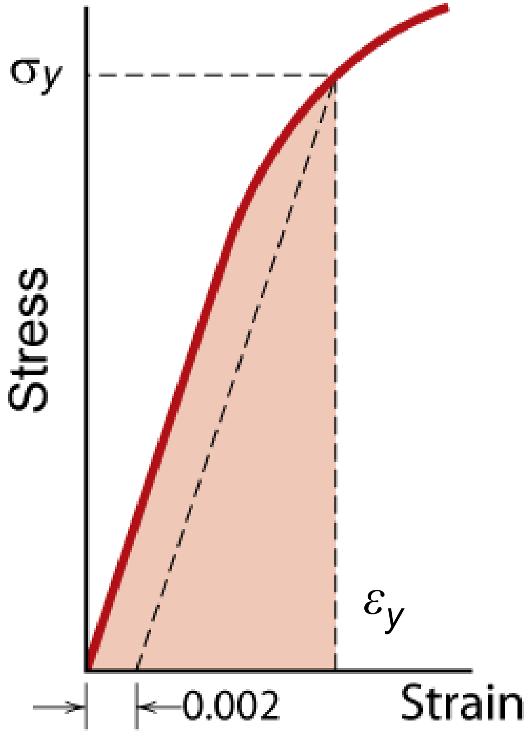
$$\%RA = \frac{A_0 - A_f}{A_0} \text{ x } 100$$



Adapted from Fig. 6.13, *Callister & Rethwisch 10e.*

Resilience

- Resilience—ability of a material to absorb energy during elastic deformation
- Energy recovered when load released
- Resilience specified by modulus of resilience, U_r



U_r = Area under stress-strain curve to yielding $= \int_0^{\varepsilon_y} \sigma \, d\varepsilon$

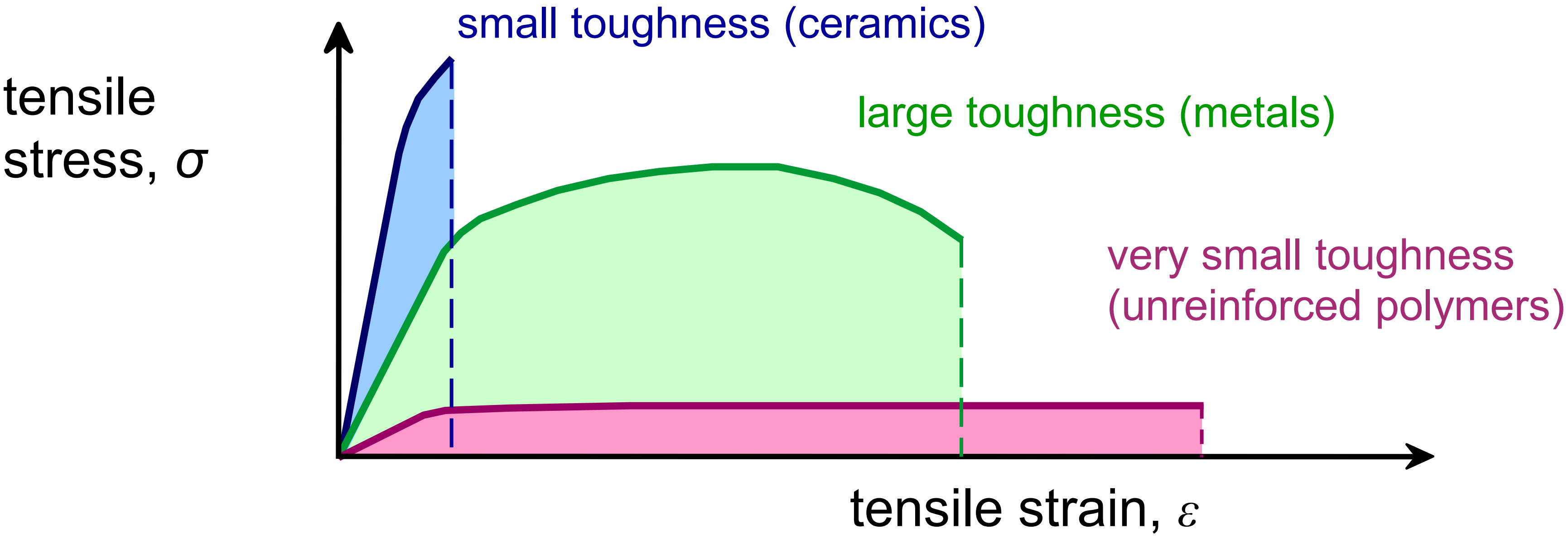
If assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2} \sigma_y \varepsilon_y$$

Fig. 6.15, *Callister & Rethwisch 10e.*

Toughness

- Toughness of a material is expressed in several contexts
- For this chapter, toughness = amount of energy absorbed before fracture
- Approximate by area under the stress-strain curve—units of energy per unit volume

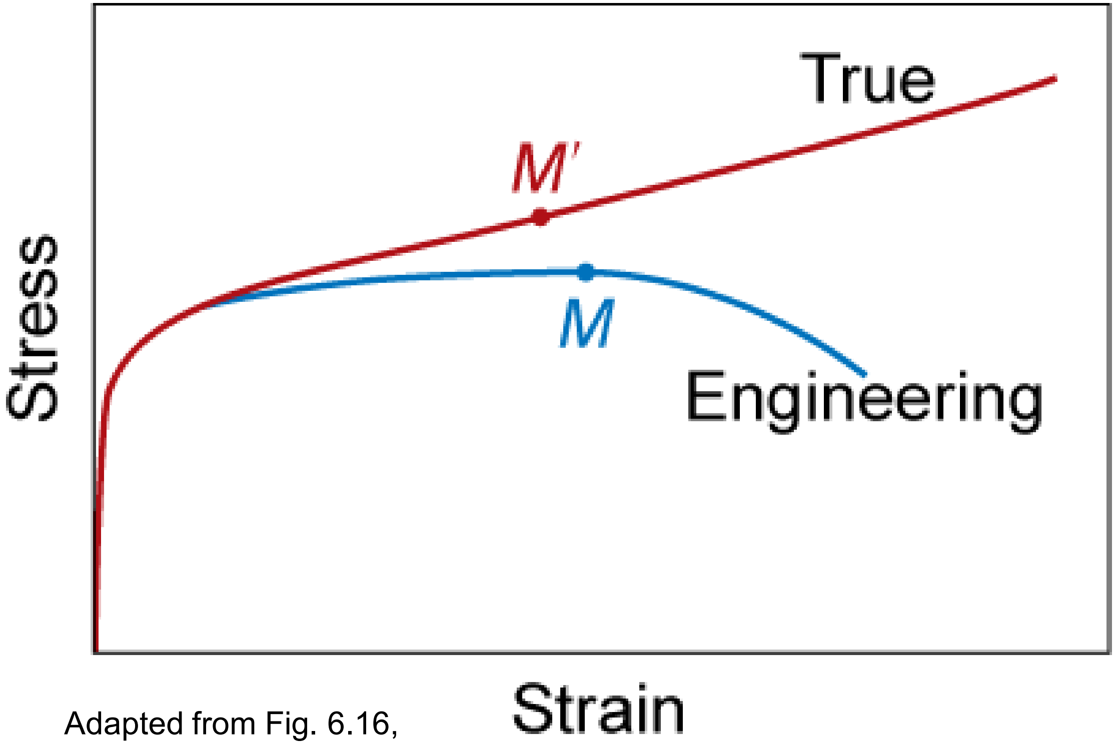


Brittle fracture: small toughness
Ductile fracture: large toughness

True Stress & Strain

- True stress $\sigma_T = F / A_i$
- True strain $\varepsilon_T = \ln\left(\ell_i / \ell_o\right)$

where A_i = instantaneous cross-sectional area



Adapted from Fig. 6.16, *Callister & Rethwisch 10e.*

Conversion Equations: valid only to the onset of necking

$$\boxed{\begin{aligned}\sigma_T &= \sigma\left(1+\varepsilon\right)\\ \varepsilon_T &= \ln\left(1+\varepsilon\right)\end{aligned}}$$

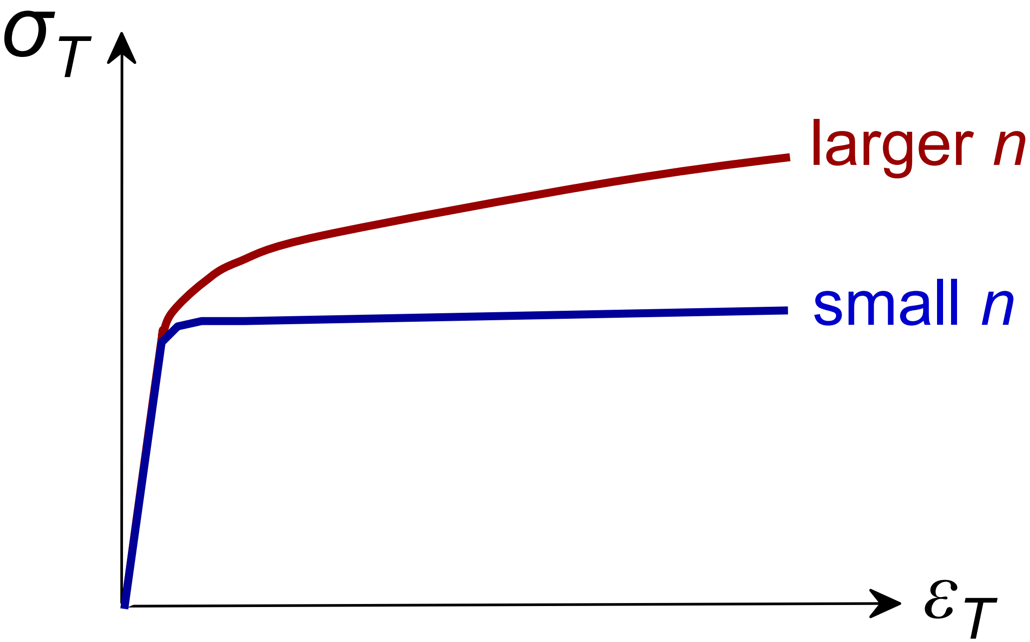
True Stress-True Strain Relationship

- Most alloys, between point of yielding and onset of necking

$$\sigma_T = K(\varepsilon_T)^n$$

  - n and K values depend on alloy and treatment
  - n = strain-hardening exponent
  - $n < 1.0$

- σ_T vs. ε_T -- influence of n.



Elastic Strain Recovery

yield strength for 2nd deformation = σ_{y_i} →

initial yield strength = σ_{y_o} →

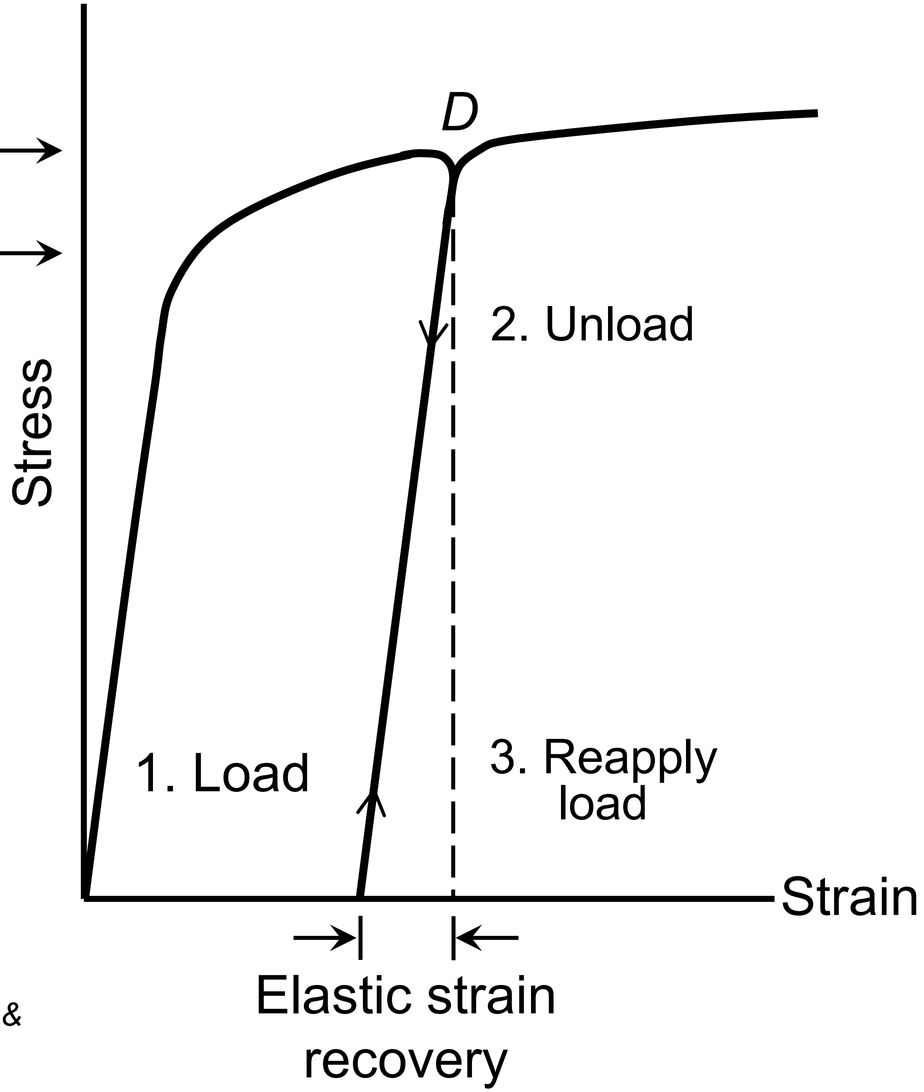
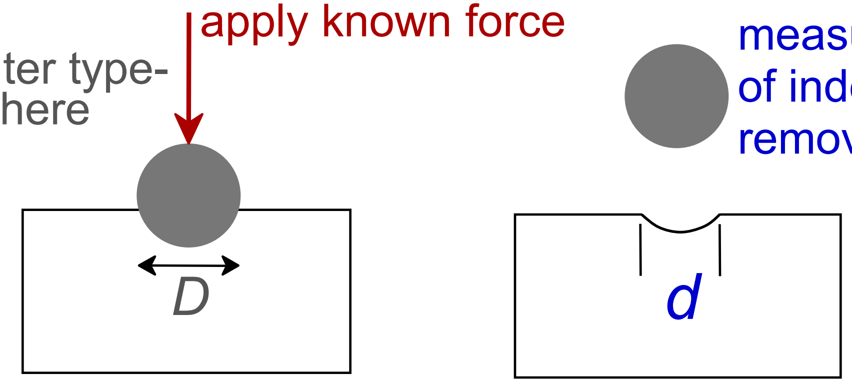


Fig. 6.17, *Callister & Rethwisch 10e.*

Hardness

- Measure of resistance to surface plastic deformation—dent or scratch.
- Large hardness means:
  - -- high resistance to deformation from compressive loads.
  - -- better wear properties.



apply known force

one indenter type-
10 mm sphere

measure size
of indent after
removing load

Smaller indents
mean larger
hardness.

most plastics

brasses Al alloys

easy to machine steels

file hard

cutting tools

nitrided steels

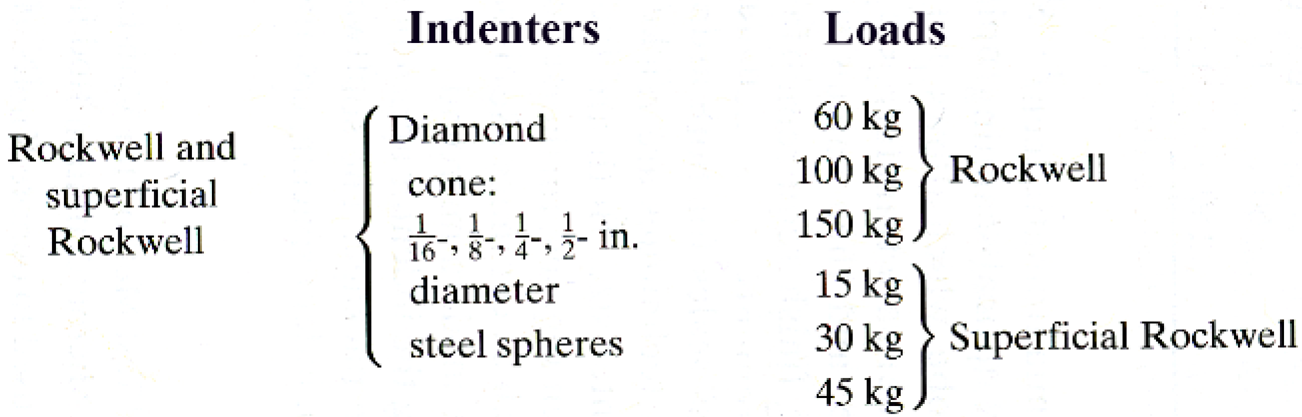
diamond

increasing hardness

Measurement of Hardness

Rockwell Hardness

- Several scales—combination of load magnitude, indenter size



Indenters **Loads**

Rockwell and superficial Rockwell

Diamond cone: $\frac{1}{16}$-, $\frac{1}{8}$-, $\frac{1}{4}$-, $\frac{1}{2}$- in. diameter steel spheres

60 kg
100 kg
150 kg
Rockwell

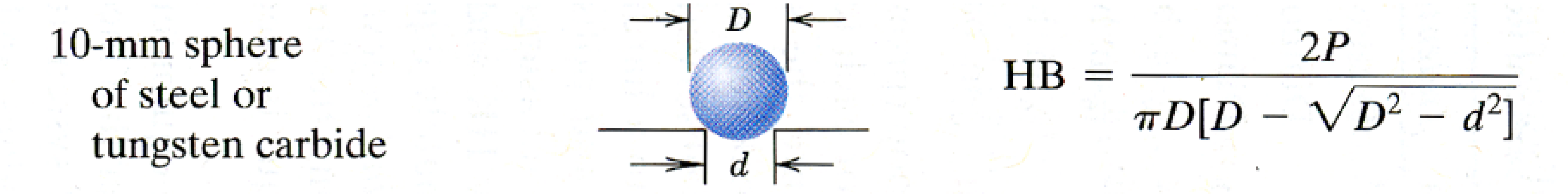
15 kg
30 kg
45 kg
Superficial Rockwell

- Examples:
  - Rockwell A Scale – 60 kg load/diamond indenter
  - Superficial Rockwell 15T Scale – 15 kg load/ 1/16 in. indenter
- Rockwell hardness designation: (hardness reading) HR
- Examples: 57 HRA; 63 HR15T
- Hardness range for each scale: 0–130 HR;
  useful range: 20–100 HR

Measurement of Hardness (cont.)

Brinell Hardness

- Single scale
- Brinell hardness designation: (hardness reading) HB



10-mm sphere
of steel or
tungsten carbide

$$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$$

  - P = load (kg)
  - 500 kg $\leq P \leq$ 3000 kg (500 kg increments)
- Relationships—Brinell hardness & tensile strength
  - TS (psia) = 500 x HB
  - TS (MPa) = 3.45 x HB

Design/Safety Factors

- Because of design uncertainties allowances must be made to protect against unanticipated failure
- For structural applications, to protect against possibility of failure—use working stress, σ_w, and a factor of safety, N

$$\sigma_w = \frac{\sigma_y}{N}$$

yield strength

Depending on application, N is between 1.2 and 4

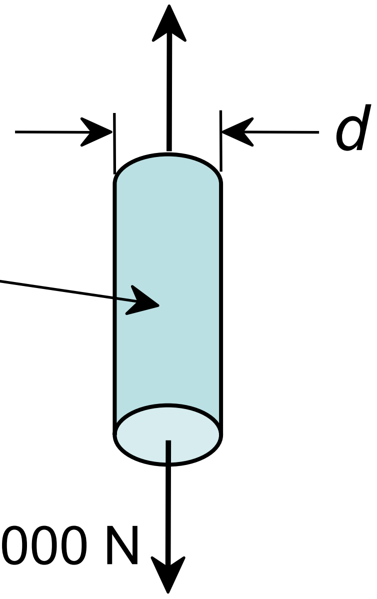
Design/Safety Factors (cont.)

Example Problem: A cylindrical rod, to be constructed from a steel that has a yield strength of 310 MPa, is to withstand a load of 220,000 N without yielding. Assuming a value of 4 for *N*, specify a suitable bar diameter.

$$\underbrace{\sigma_w}_{\dfrac{220{,}000\text{ N}}{\pi\left(\dfrac{d}{2}\right)^2}} = \frac{\sigma_y}{\underbrace{N}_{4}}$$

Steel rod:

σ_y = 310 MPa



Solving for the rod diameter *d* yields

$$\boxed{d = 0.060\text{ m} = 60\text{ mm}}$$

Summary

- Applied mechanical force—normalized to stress
- Degree of deformation—normalized to strain
- Elastic deformation:
  - ––non-permanent; occurs at low levels of stress
  - ––stress-strain behavior is linear
- Plastic deformation
  - ––permanent; occurs at higher levels of stress
  - ––stress-strain behavior is nonlinear
- Stiffness—a material's resistance to elastic deformation
  - ––elastic (or Young's) modulus

Summary (cont.)

- Strength—a material's resistance to plastic deformation
  ––yield and tensile strengths
- Ductility—amount of plastic deformation at failure
  ––percents elongation, reduction in area
- Hardness—resistance to localized surface deformation & compressive stresses
  ––Rockwell, Brinell hardnesses

Dislocations & Strengthening Mechanisms

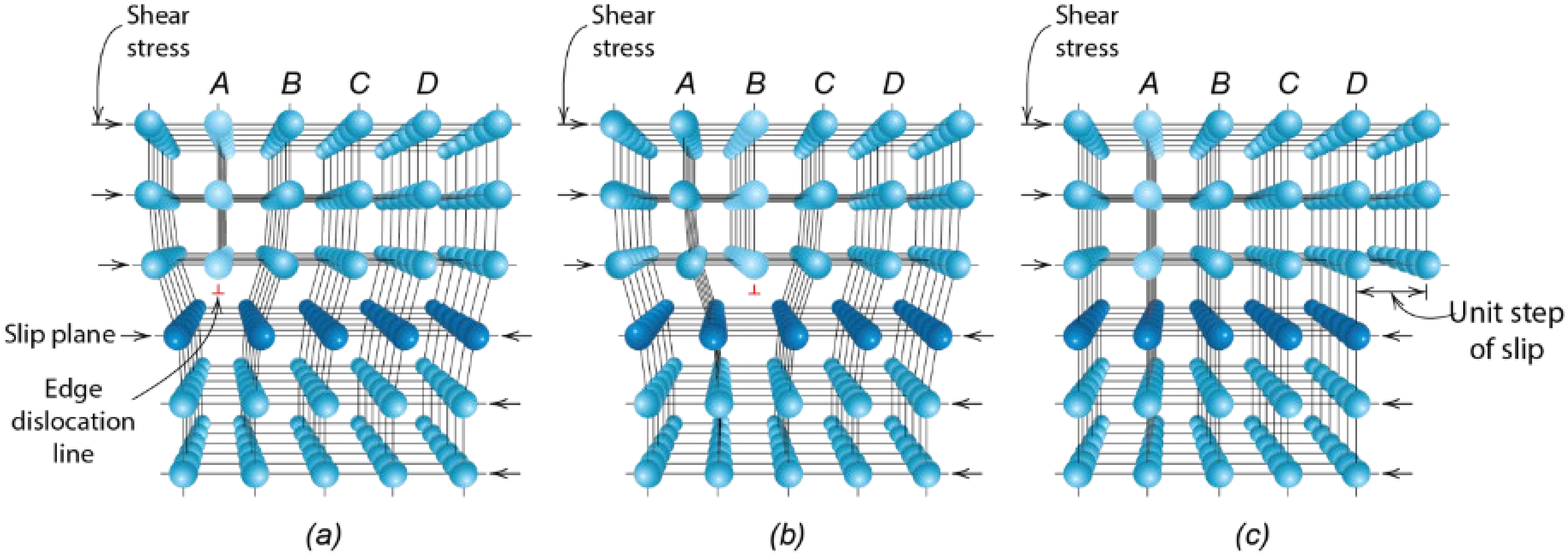
ISSUES TO ADDRESS...

- How are dislocations involved in the plastic deformation of metals/metal alloys?
- Does the crystal structure of a metal affect its mechanical characteristics? If so, how and why?
- How are mechanical properties affected by dislocation mobilities?
- What techniques are used to increase the strength/hardness of metals/alloys?
- How are mechanical characteristics of deformed metal specimens altered by heat treatments?

Plastic Deformation by Dislocation Motion

- Plastic deformation occurs by motion of dislocations (edge, screw, mixed) – process called slip
- Applied shear stress can cause extra half-plane of atoms [and edge dislocation line (⊥)] to move as follows:

Fig. 7.1, *Callister & Rethwisch 10e.*



- Atomic bonds broken and reformed along slip plane as dislocation (extra half plane) moves.

Analogy Between Dislocation Motion and Caterpillar Locomotion

- Caterpillar locomotion – hump formed and propelled by lifting and shifting of leg pairs
- Dislocation motion – movement of extra half-plane of atoms by breaking and reforming of interatomic bonds

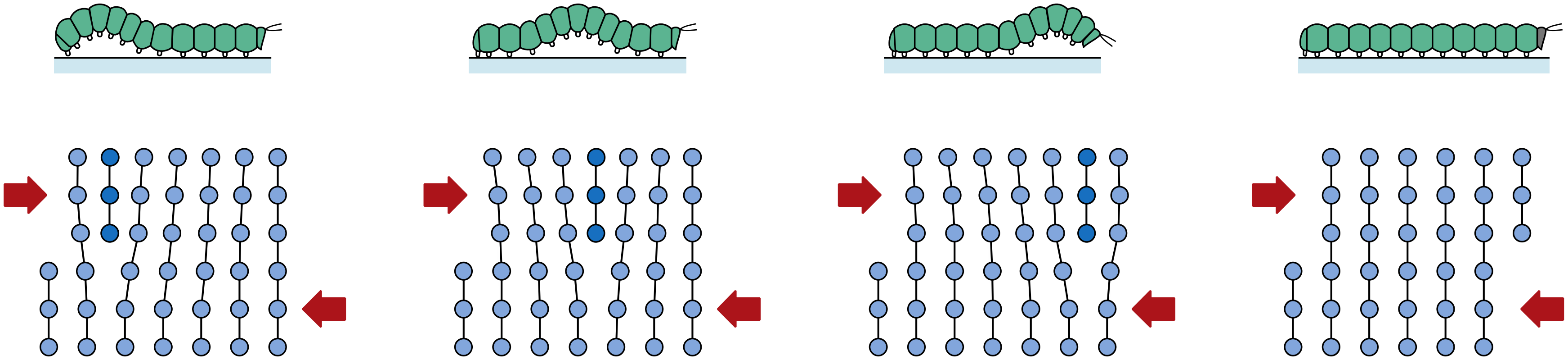
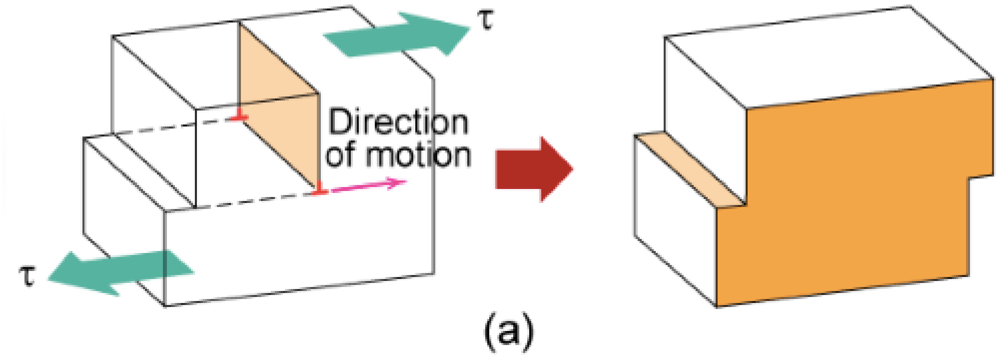


Fig. 7.3, *Callister & Rethwisch 10e.*

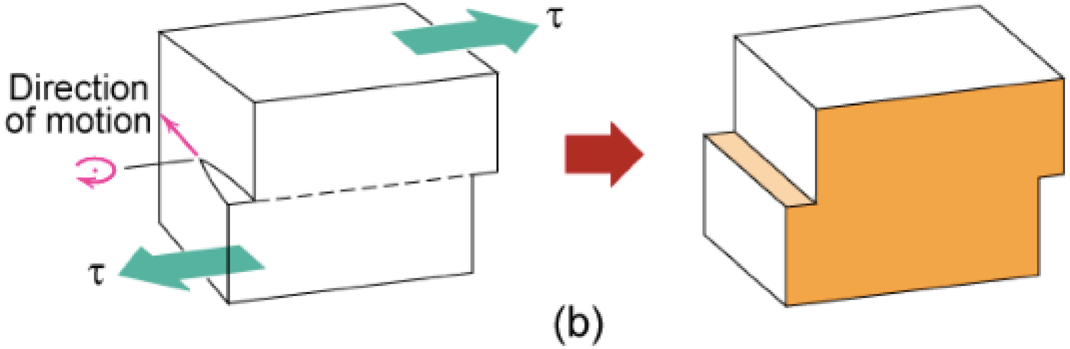
Motion of Edge and Screw Dislocations

- Direction of edge disl. line (⊥) motion—in direction of applied shear stress τ.



Edge dislocation

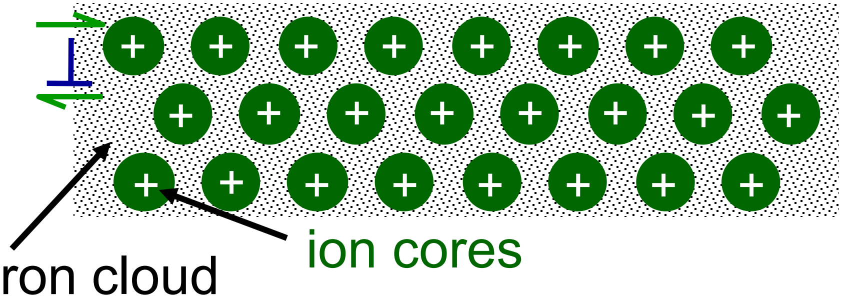
- Direction of screw disl. line (↺) motion—perpendicular to direction of applied shear stress.



Screw dislocation

Fig. 7.2, *Callister & Rethwisch 10e.*

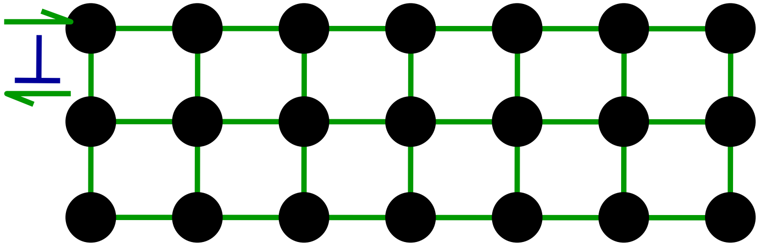
Dislocation Characteristics Metals



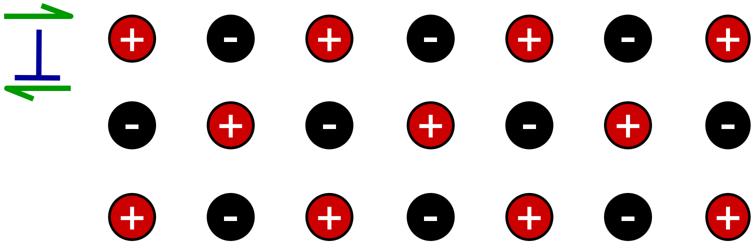
- Metals:
    - Examples: copper, aluminum, iron
    - Dislocation motion—relatively easy
    - Metallic bonding—non-directional
    - Close-packed planes and directions for slip

Dislocation Characteristics Ceramics

- Ceramics—Covalently Bonded
  - Examples: silicon, diamond
  - Dislocation motion—relatively difficult
  - Covalent bonding—directional



- Ceramics—Ionically Bonded
  - Examples: NaCl, MgO
  - Dislocation motion—relatively difficult
  - Few slip systems
    - motion of nearby ions of like charge (+ and -) restricted by electrostatic repulsive forces



Slip Systems

Slip System—Combination of slip plane and slip direction

- Slip Plane
  - Crystallographic plane on which slip occurs most easily
  - Plane with high planar density
- Slip Direction
  - Crystallographic direction along which slip occurs most easily
  - Direction with high linear density

Slip Systems (cont.)

- For FCC crystal structure – slip system is $\{111\}\langle 110\rangle$
    - Dislocation motion on $\{111\}$ planes
    - Dislocation motion in $\langle 110\rangle$ directions
    - A total of 12 independent slip systems for FCC

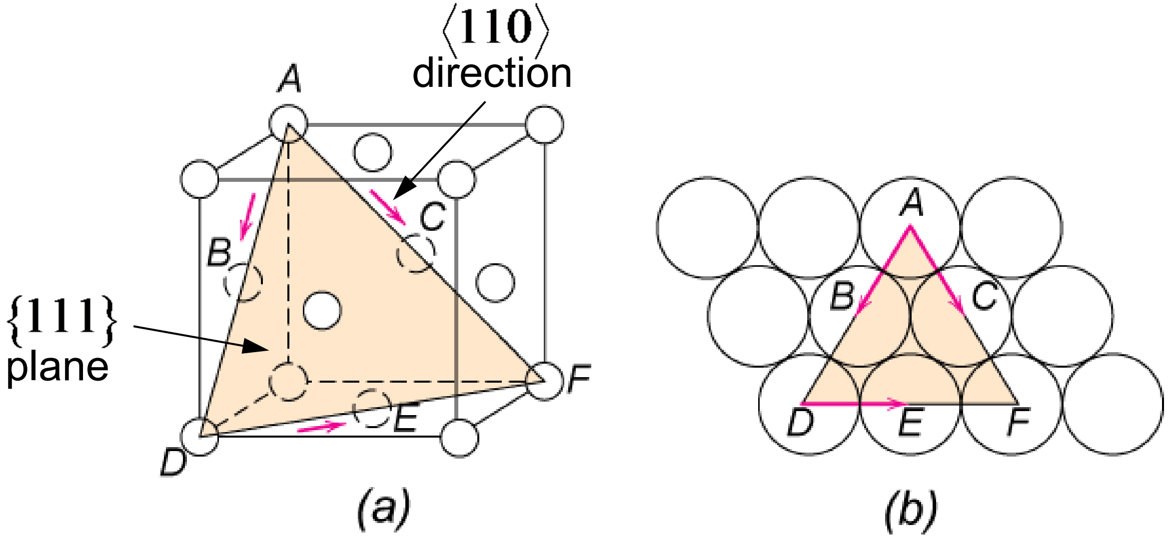


Fig. 7.6, *Callister & Rethwisch 10e.*

- For BCC and HCP— other slip systems

Slip in Single Crystals Resolved Shear Stress

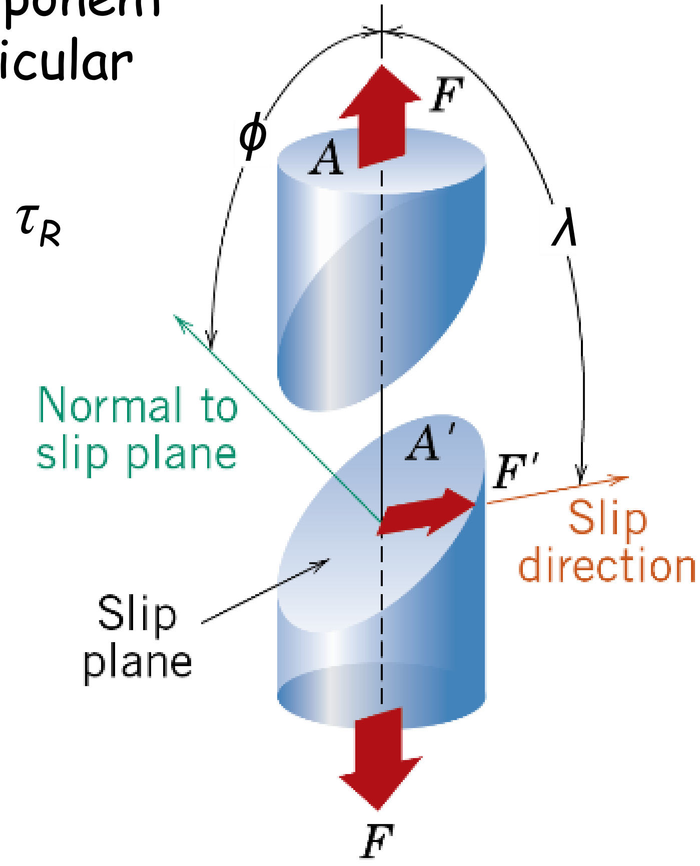
- Applied tensile stress—shear stress component when slip plane oriented neither perpendicular nor parallel to stress direction

  -- From figure, resolved shear stress, τ_R

$$\tau_R = \frac{F'}{A'}$$

- τ_R depends on orientation of normal to slip plane and slip direction with direction of tensile force F:

$$F' = F \cos \lambda \qquad A' = \frac{A}{\cos \phi}$$

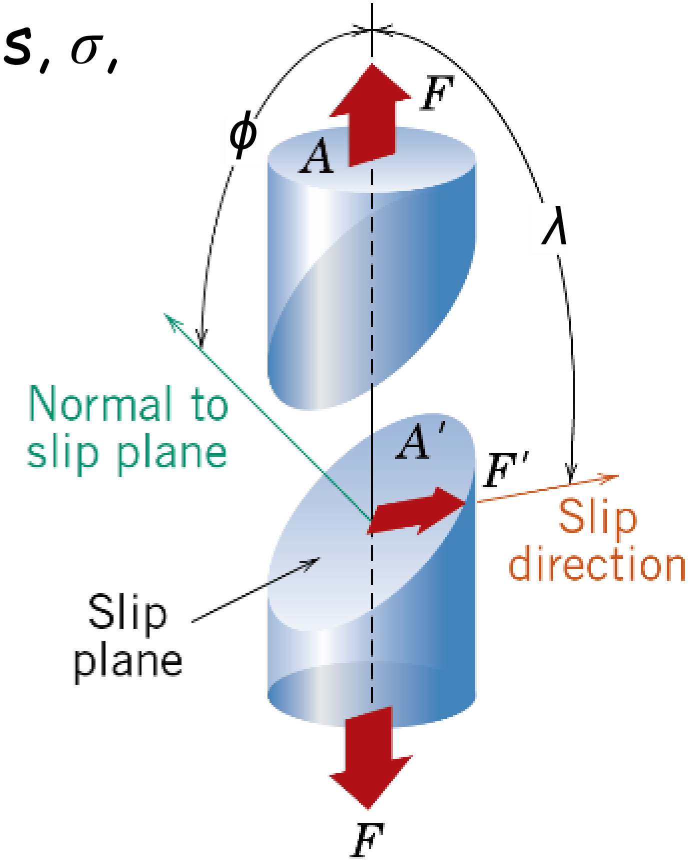


Slip in Single Crystals - Resolved Shear Stress (cont.)

- Relationship between tensile stress, σ, and τ_R:

$$\tau_R = \frac{F'}{A'} = \frac{F\cos\lambda}{\frac{A}{\cos\phi}} = \frac{F}{A}\cos\lambda\cos\phi$$

$$= \sigma\cos\lambda\cos\phi$$



Slip in Single Crystals: Critical Resolved Shear Stress

- Dislocation motion—on specific slip system—when τ_R reaches critical value:
  - -- "Critical resolved shear stress", τ_{CRSS}
  - -- Slip occurs when $\tau_R > \tau_{\text{CRSS}}$
  - -- Typically 0.1 MPa < τ_{CRSS} < 10 MPa
- In a single crystal there are
  - -- multiple slip systems
  - -- a variety of orientations
- One slip system for which τ_R is highest: $\tau_R(\text{max}) > \sigma\,(\cos\lambda\,\cos\phi\,)_{\text{max}}$
  - -- Most favorably oriented slip system
- Yield strength of single crystal, σ_y, when

$$\sigma_y = \frac{\tau_{\text{CRSS}}}{(\cos\lambda\cos\phi)_{\text{max}}}$$

Single Crystals Slip—Macroscopic Scale

- Parallel slip steps form on surface of single crystal
- Steps result from motion of large numbers of dislocations on same slip plane
- Sometimes on single crystals appear as "slip lines" (see photograph)

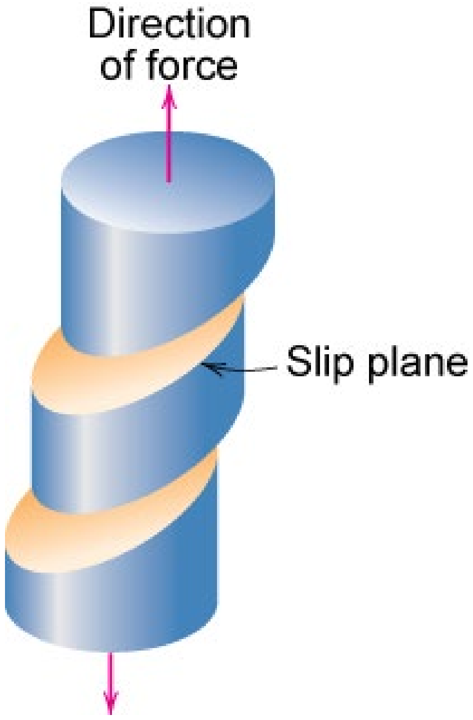


Fig. 7.8, *Callister & Rethwisch 10e.*

Deformation of Single Crystals - Example Problem

A single crystal of some metal has a τ_{crss} of 20.7 MPa and is exposed to a tensile stress of 45 MPa.

(a) Will yielding occur when $\phi = 60°$ and $\lambda = 35°$?

(b) If not, what stress is necessary?

Solution:

(a) First calculate τ_R

$$\tau_R = \sigma \cos\lambda \cos\phi$$

$$\tau_R = (45\ \text{MPa})\left[\cos(35°)\cos(60°)\right]$$

$$= 18.4\ \text{MPa}$$

Since $\tau_R (18.4\ \text{MPa}) < \tau_{crss} (20.7\ \text{MPa})$ -- no yielding

Deformation of Single Crystals - Example Problem (cont.)

(b) To calculate the required tensile stress to cause yielding use the equation:

$$\sigma_y = \frac{\tau_{\mathrm{CRSS}}}{\cos\lambda\cos\phi}$$

With specified values

$$\sigma_y = \frac{20.7\ \mathrm{MPa}}{\cos(35°)\cos(60°)}$$

$$= 50.5\ \mathrm{MPa}$$

Therefore, to cause yielding, $\boxed{\sigma \geq 50.5\ \mathrm{MPa}}$

Slip in Polycrystalline Materials

- Polycrystalline materials—many grains, often random crystallographic orientations

- Orientation of slip planes, slip directions (ϕ, λ)—vary from grain to grain.

- On application of stress—slip in each grain on most favorable slip system.
  - with largest τ_R
  - when $\tau_R > \tau_{crss}$

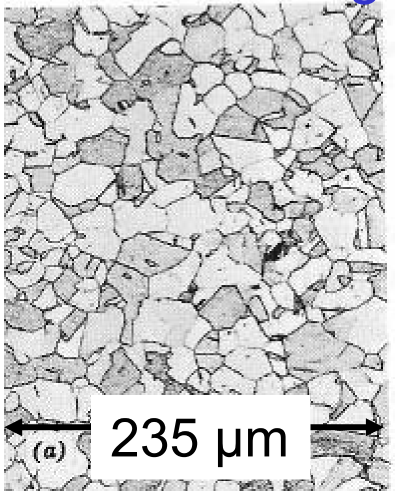
- In photomicrograph—note slip lines in grains have different orientations.

Adapted from Fig. 7.10, *Callister & Rethwisch 10e.* (Photomicrograph courtesy of C. Brady, National Bureau of Standards [now the National Institute of Standards and Technology, Gaithersburg, MD].)

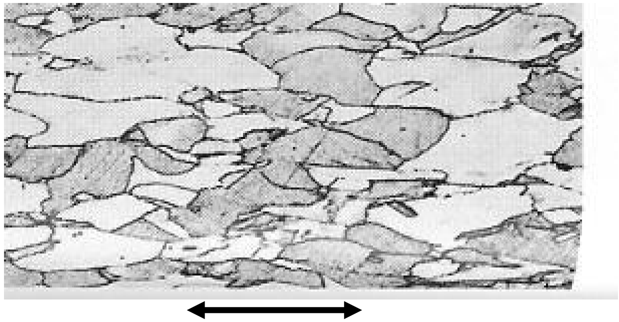
Slip in Polycrystalline Materials (cont.)

- Grains change shape (become distorted)—during plastic deformation—due to slip
- Manner of grain distortion similar to gross plastic deformation
  - Grain structures before and after deformation (from rolling)
  - Before rolling—grains equiaxed & randomly oriented
    - Properties isotropic
  - After rolling (deformation)—grains elongated in rolling direction
    - Also preferred crystallographic orientation of grains
    - Properties become somewhat anisotropic

- before rolling



- after rolling



Adapted from Fig. 7.11, *Callister & Rethwisch 10e.* (from W.G. Moffatt, G.W. Pearsall, and J. Wulff, *The Structure and Properties of Materials*, Vol. I, *Structure*, p. 140, John Wiley and Sons, New York, 1964.)

Strengthening Mechanisms for Metals

- For a metal to plastically deform—dislocations must move
- Strength and hardness—related to mobility of dislocations
  - -- Reduce disl. mobility—metal strengthens/hardens
  - -- Greater forces necessary to cause disl. motion
  - -- Increase disl. mobility—metal becomes weaker/softer
- Mechanisms for strengthening/hardening metals—decrease disl. mobility
- 3 mechanisms discussed
  - -- Grain size reduction
  - -- Solid solution strengthening
  - -- Strain hardening (cold working)

Strengthening Mechanisms for Metals I – Reduce Grain Size

- Grain boundaries act as barriers to dislocation motion
- At boundary
  - — Slip planes change directions (note in illustration)
  - — Discontinuity of slip planes

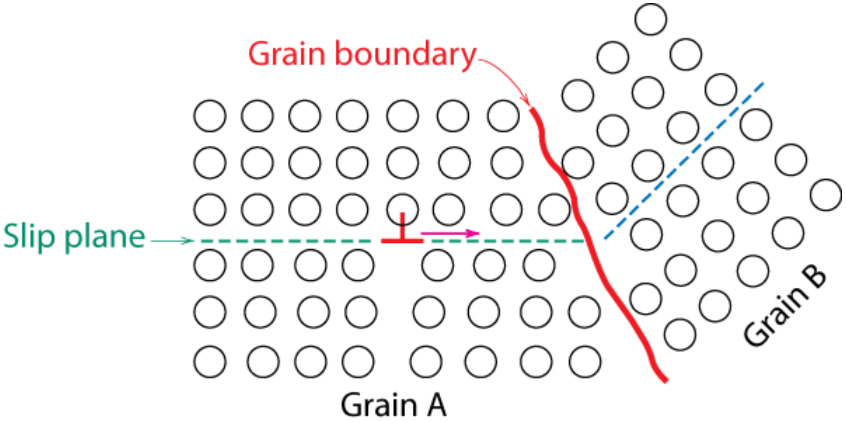


Fig. 7.14, *Callister & Rethwisch 10e.* (From L. H. Van Vlack*, A Textbook of Materials Technology*, Addison-Wesley Publishing Co., 1973. Reproduced with the permission of the Estate of Lawrence H. Van Vlack.)

- Reduce grain size
  - — increase grain boundary area
  - — more barriers to dislocation motion
  - — increase yield strength, tensile strength & hardness
- Dependence of σ_y on average grain diameter, *d:*

$$\sigma_{yield} = \sigma_0 + k_y d^{-1/2}$$

—σ_0, k_y = material constants

Strength. for Metals II – Solid-Solution Strengthening

- Lattice strains around dislocations
  - Illustration notes locations of tensile, compressive strains around an edge dislocation

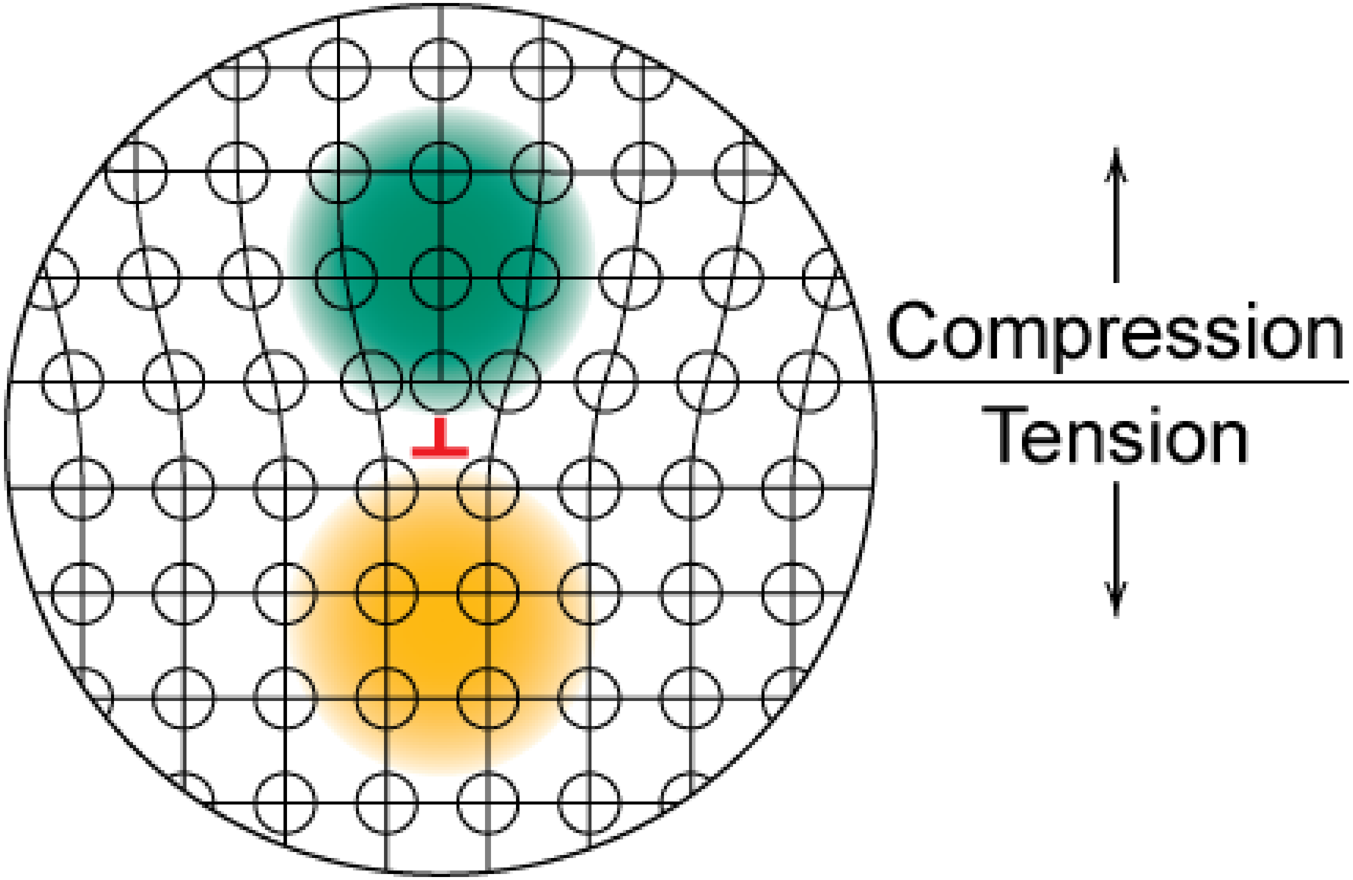


Fig. 7.4, *Callister & Rethwisch 10e.* (Adapted from W.G. Moffatt, G.W. Pearsall, and J. Wulff, *The Structure and Properties of Materials*, Vol. I, *Structure*, p. 140, John Wiley and Sons, New York, 1964.)

Solid Solution Strengthening (cont.)

- Lattice strain interactions with strains introduced by impurity atoms
- Small substitutional impurities introduce tensile strains
- When located above slip line for edge dislocation as shown:
  - partial cancellation of impurity (tensile) and disl. (compressive) strains
  - higher shear stress required to cause disl. motion

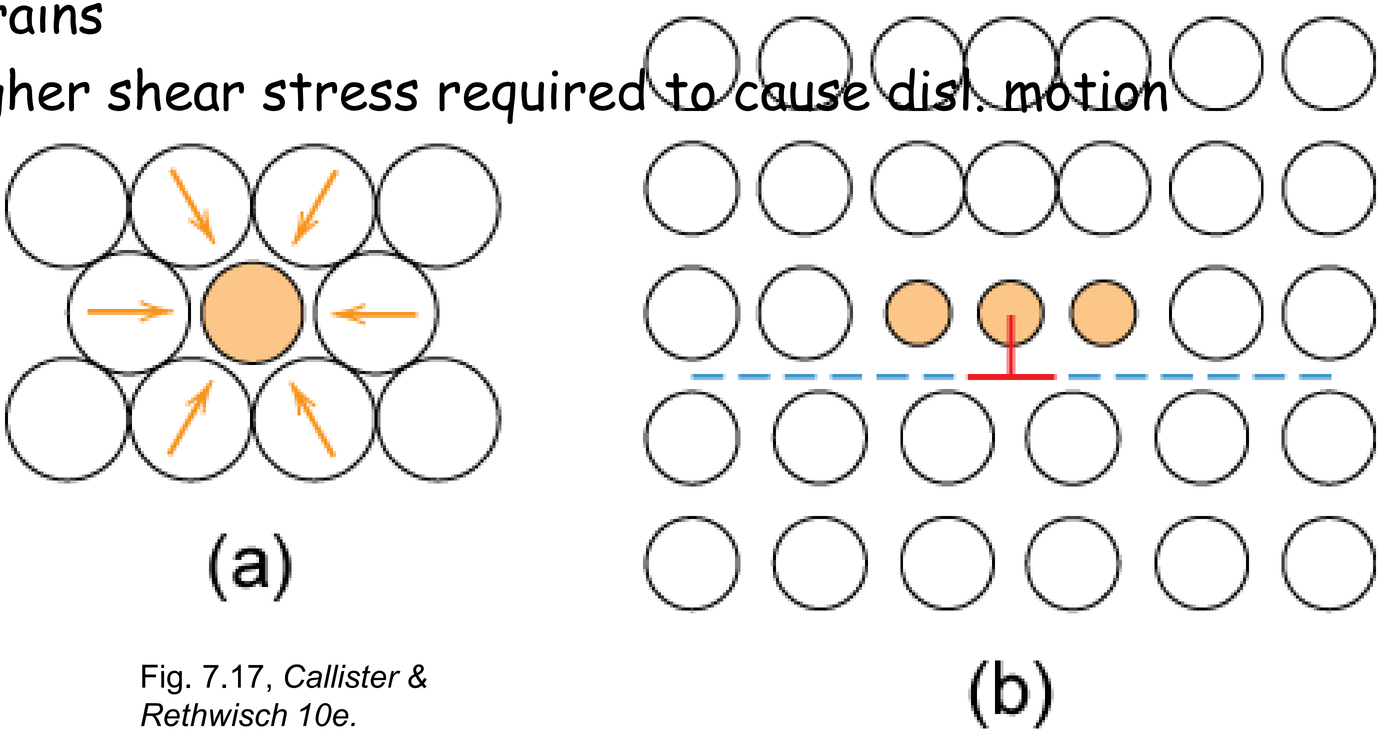


Fig. 7.17, *Callister & Rethwisch 10e.*

Solid Solution Strengthening (cont.)

- Large substitutional impurities introduce compressive strains
- When located below slip line for edge dislocation as shown:
  - partial cancellation of impurity (compressive) and disl. (tensile) strains
  - higher shear stress required to cause disl. motion

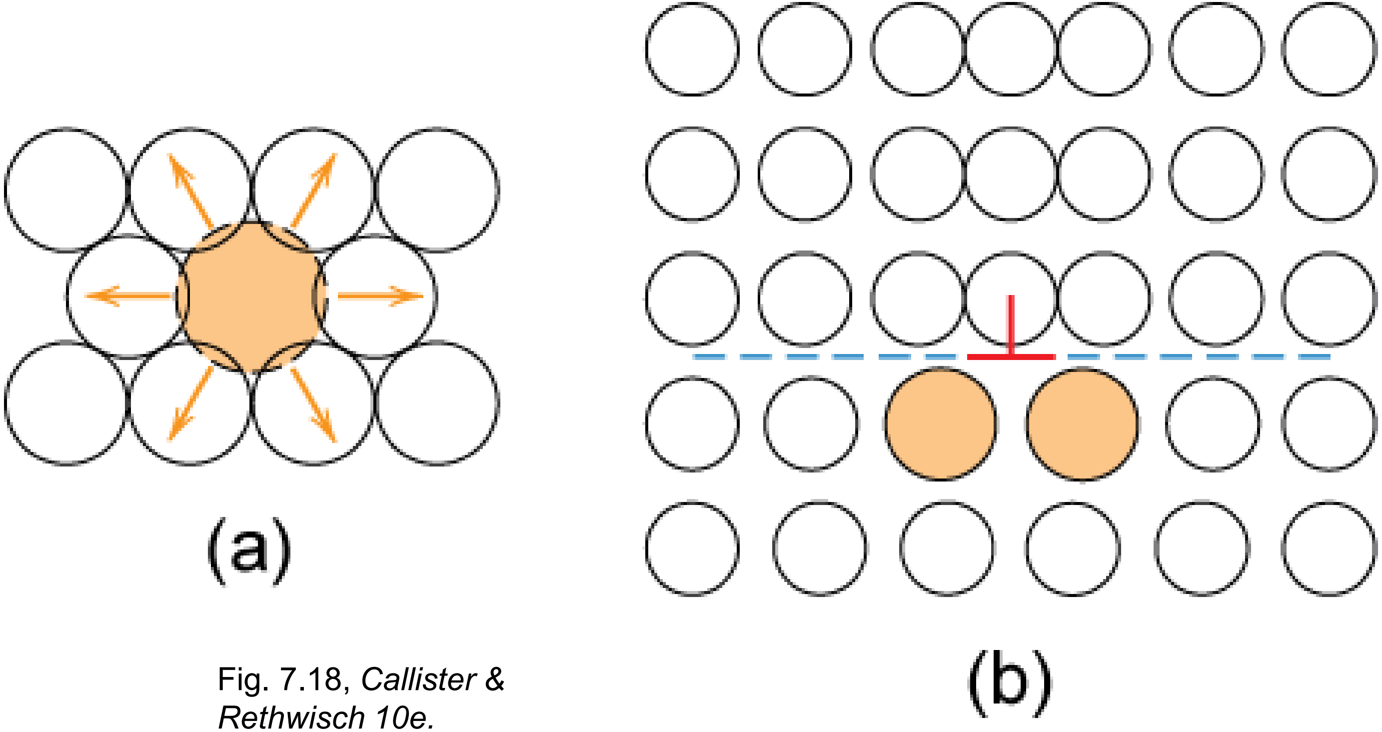
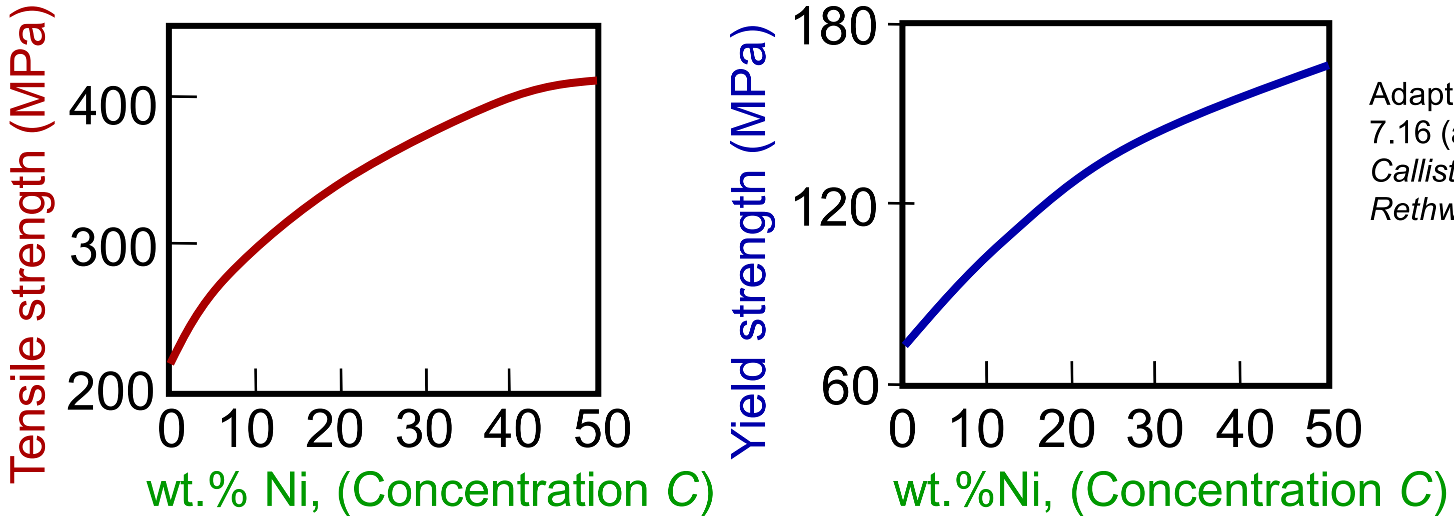


Fig. 7.18, *Callister & Rethwisch 10e.*

Solid Solution Strengthening (cont.)

- Alloying Cu with Ni increases σ_y and *TS*.
- Tensile strength & yield strength increase with wt% Ni.



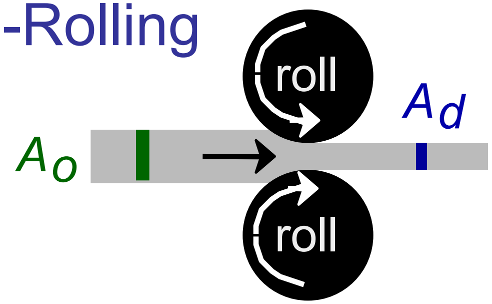
Adapted from Fig. 7.16 (a) and (b), *Callister & Rethwisch 10e.*

- Empirically, $\sigma_y \propto C^{1/2}$

Strengthening for Metals III - Strain Hardening

- Plastically deforming most metals at room temp. makes them harder and stronger
- Phenomenon called "Strain hardening (or cold working)”
- Deformation—often reduction in cross-sectional area.

-Rolling



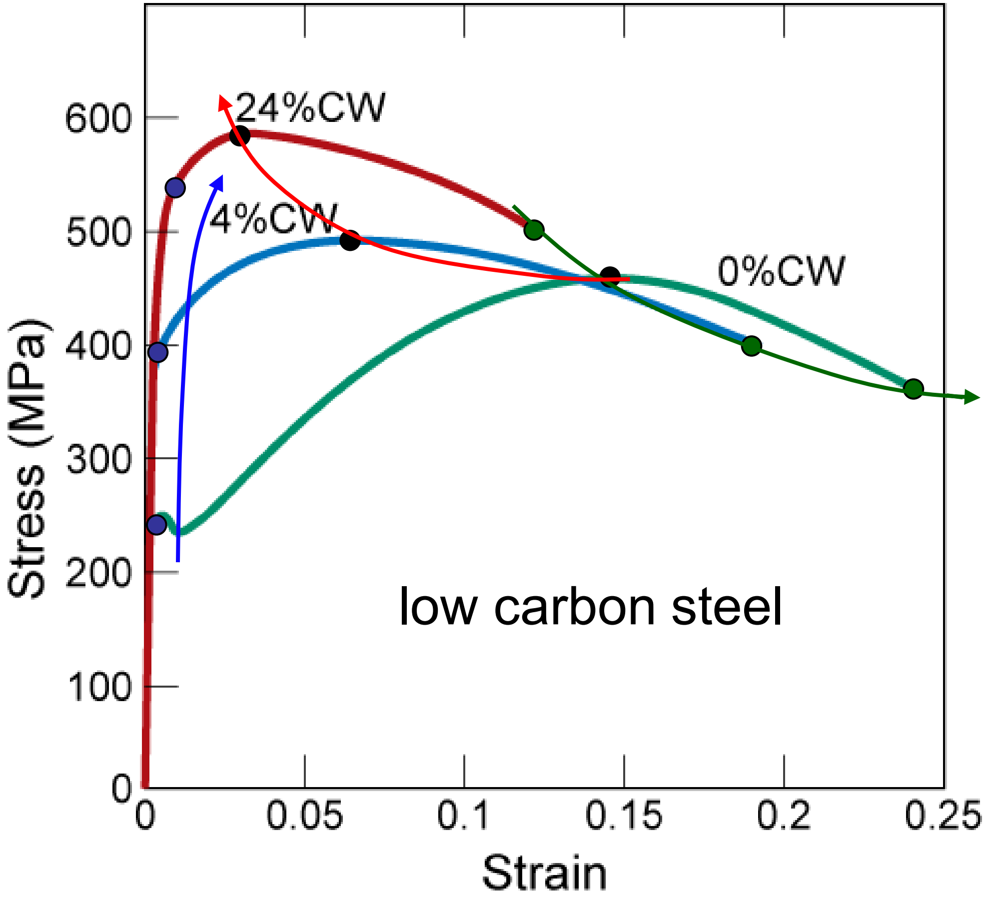
- Deformation amt. = percent coldwork (%CW)

$$\%CW = \frac{A_o - A_d}{A_o} \times 100$$

Strain Hardening (cont.)

As %CW increases

- Yield strength (σ_y) increases.
- Tensile strength (*TS*) increases.
- Ductility (%*EL* or %*AR*) decreases.

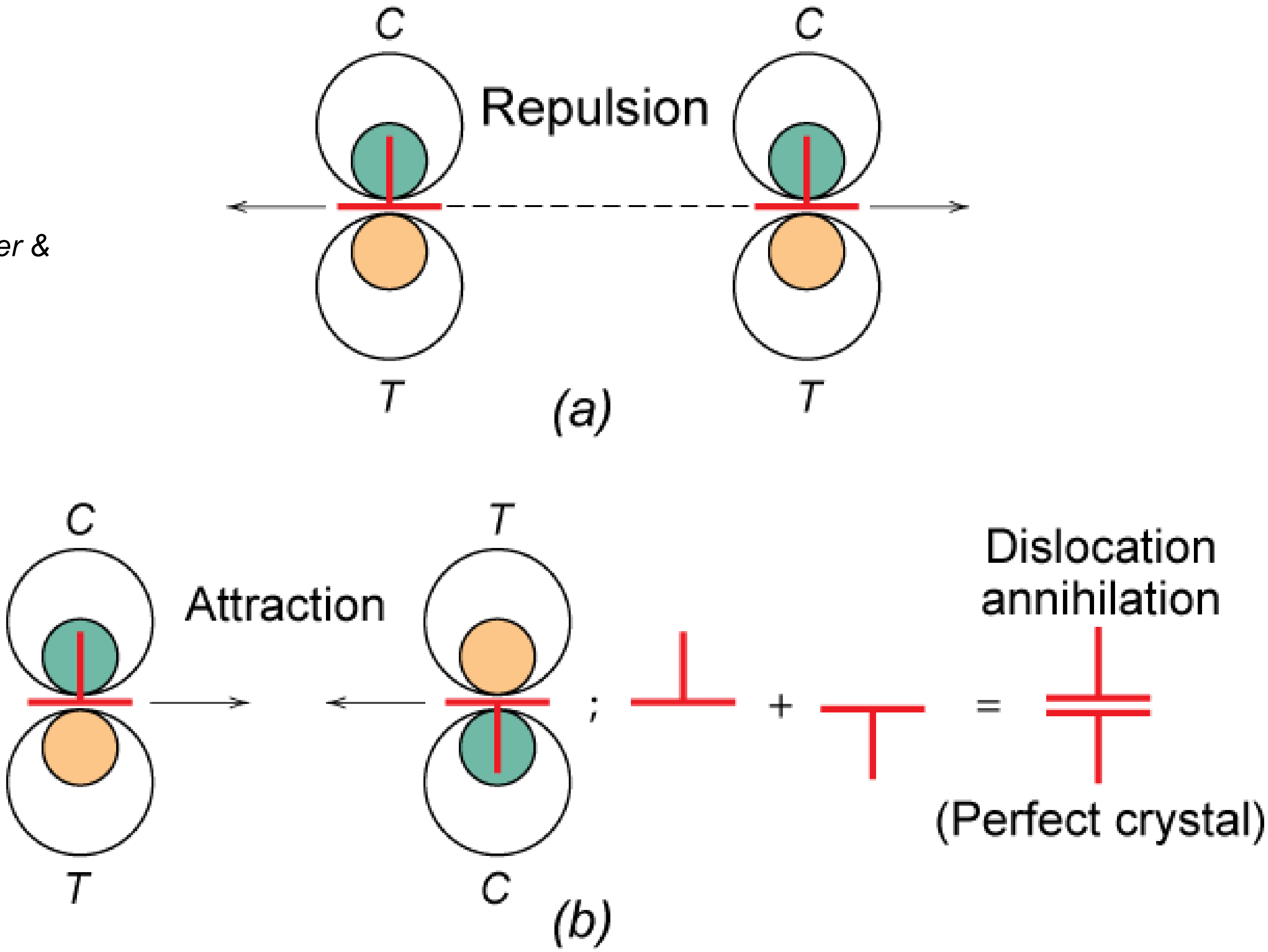


Adapted from Fig. 7.20,
Callister & Rethwisch 10e.

Strain Hardening (cont.)

Lattice strain interactions between dislocations

Fig. 7.5, *Callister & Rethwisch 10e.*



Strain Hardening (cont.) - Dislocation Density and Cold Working

$$\text{Dislocation density} = \frac{\text{total dislocation length}}{\text{unit volume}}$$

- Dislocation density in undeformed metal

  → 10^5-10^6 mm^{-2}

- Dislocation density increases with increasing deformation

- Dislocation density in deformed (cold-worked) metal

  → 10^9-10^{10} mm^{-2}

Strain Hardening (cont.)- Mechanism of Strain Hardening

- Dislocation structure in Ti after cold working.

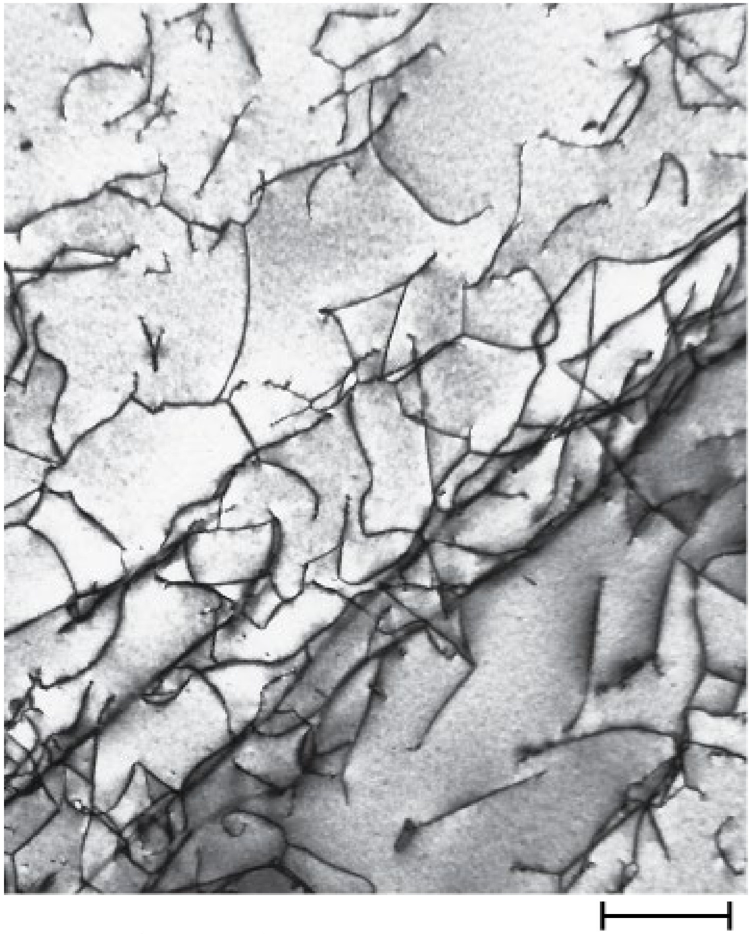


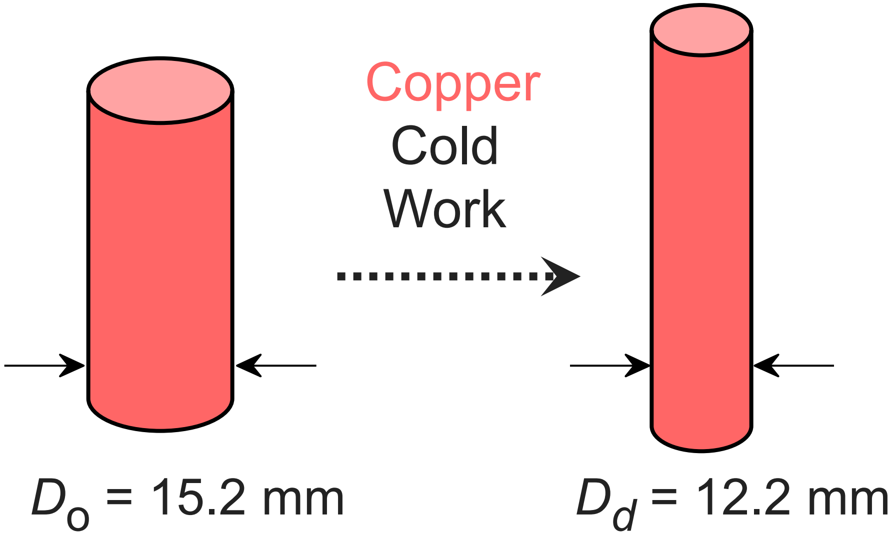
Fig. 4.7, *Callister & Rethwisch 10e.* (Courtesy of M.R. Plichta, Michigan Technological University.)

- Dislocation density increases with deformation (cold work) by formation of new dislocations
- As dislocation density increases, distance between dislocations decreases
- On average, disl.-disl. strain interactions are repulsive
- Dislocation motion hindered by presence of other dislocations

Affect of Cold Work on Mechanical Properties

Example Problem:

Compute the yield and tensile strengths, and ductility for a cylindrical Cu specimen that has been cold worked by reducing its diameter from 15.2 mm to 12.2 mm.



Example Problem (cont.)

- Solution:

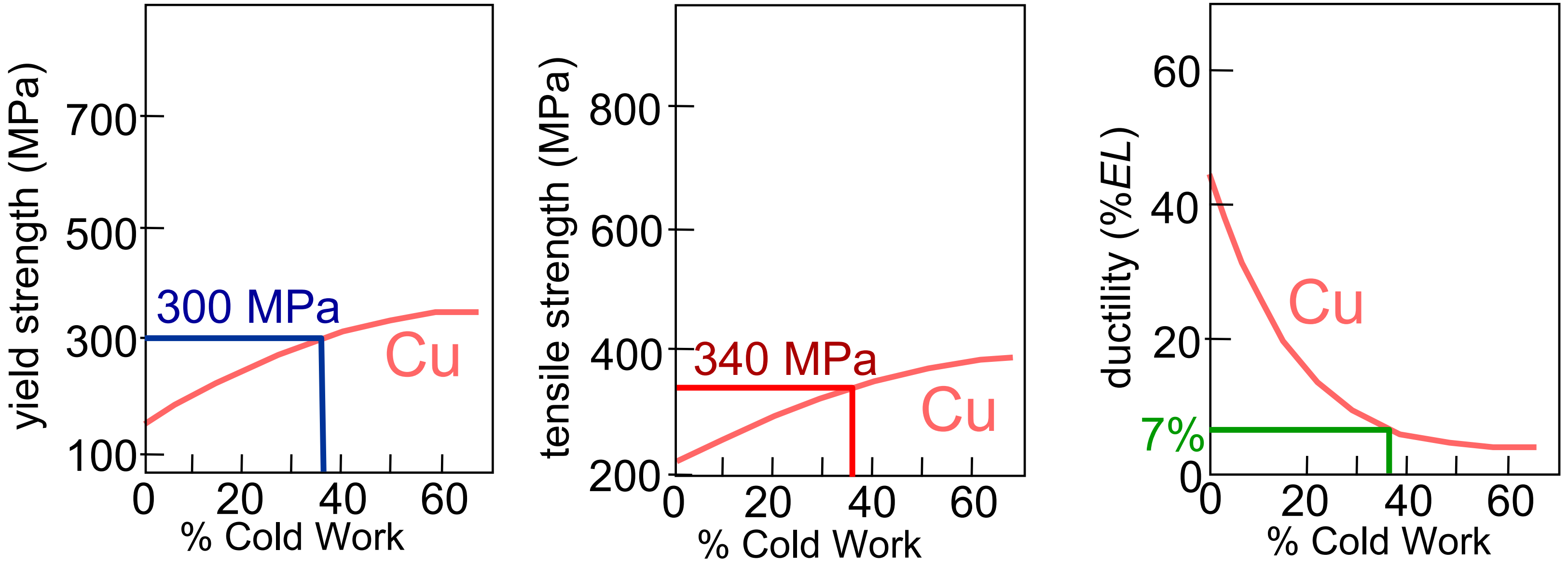
$$\%CW = \frac{\cancel{\pi}\left(\frac{D_o}{\cancel{2}}\right)^2 - \cancel{\pi}\left(\frac{D_d}{\cancel{2}}\right)^2}{\cancel{\pi}\left(\frac{D_o}{\cancel{2}}\right)^2} \text{ x } 100$$

$$= \frac{D_o^2 - D_d^2}{D_o^2} \text{ x } 100$$

$$\%CW = \frac{(15.2 \text{ mm})^2 - (12.2 \text{ mm})^2}{(15.2 \text{ mm})^2} \text{ x } 100 = \boxed{35.6\%}$$

Example Problem (cont.)

- Yield and tensile strength, and ductility (%EL) are determined graphically as shown below for %CW = 35.6%



σ_y = 300 MPa

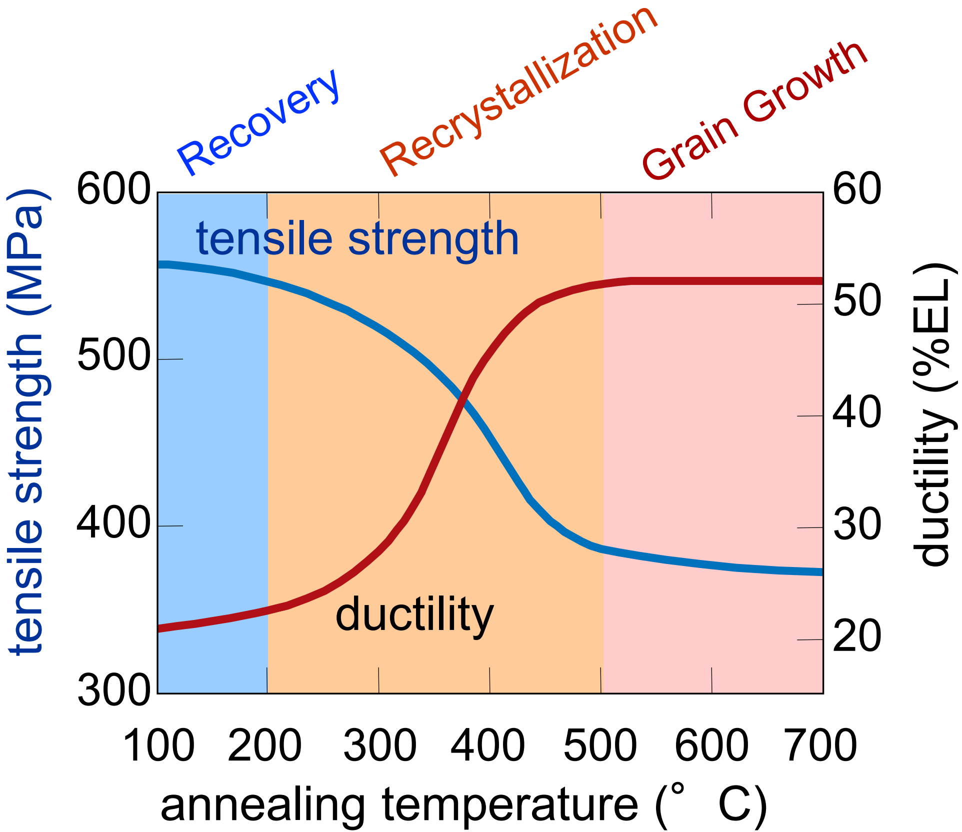
TS = 340 MPa

%*EL* = 7%

Fig. 7.19, *Callister & Rethwisch 10e.* [Adapted from *Metals Handbook: Properties and Selection: Irons and Steels*, Vol. 1, 9th edition, B. Bardes (Editor), 1978; and *Metals Handbook: Properties and Selection: Nonferrous Alloys and Pure Metals*, Vol. 2, 9th edition, H. Baker (Managing Editor), 1979. Reproduced by permission of ASM International, Materials Park, OH.]

Heat Treatment of Cold-Worked Metal Alloys

- Heat treating cold worked metals brings about changes in structure and properties
- As a result, effects of cold work are nullified!
- This type of heat treatment sometimes termed "annealing"
- 1 hour treatment at T_{anneal} decreases tensile strength & increases %EL



Three Annealing stages:

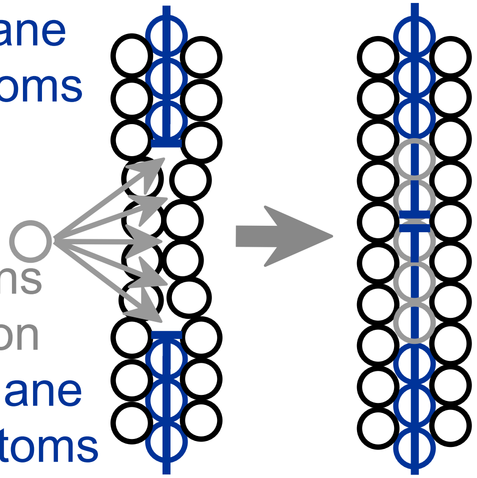
1. Recovery (100-200° C)
2. Recrystallization (200-500° C)
3. Grain Growth (> 500° C)

Fig. 7.22, *Callister & Rethwisch 10e.* (Adapted from G. Sachs and K. R. Van Horn, *Practical Metallurgy, Applied Metallurgy and the Industrial Processing of Ferrous and Nonferrous Metals and Alloys*, 1940. Reproduced by permission of ASM International, Materials Park, OH.)

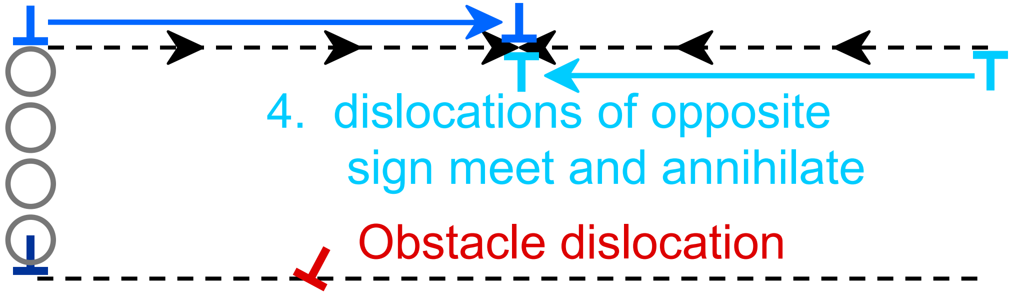
Recovery

During recovery – reduction in disl. density – annihilation of disl.

- Scenario 1

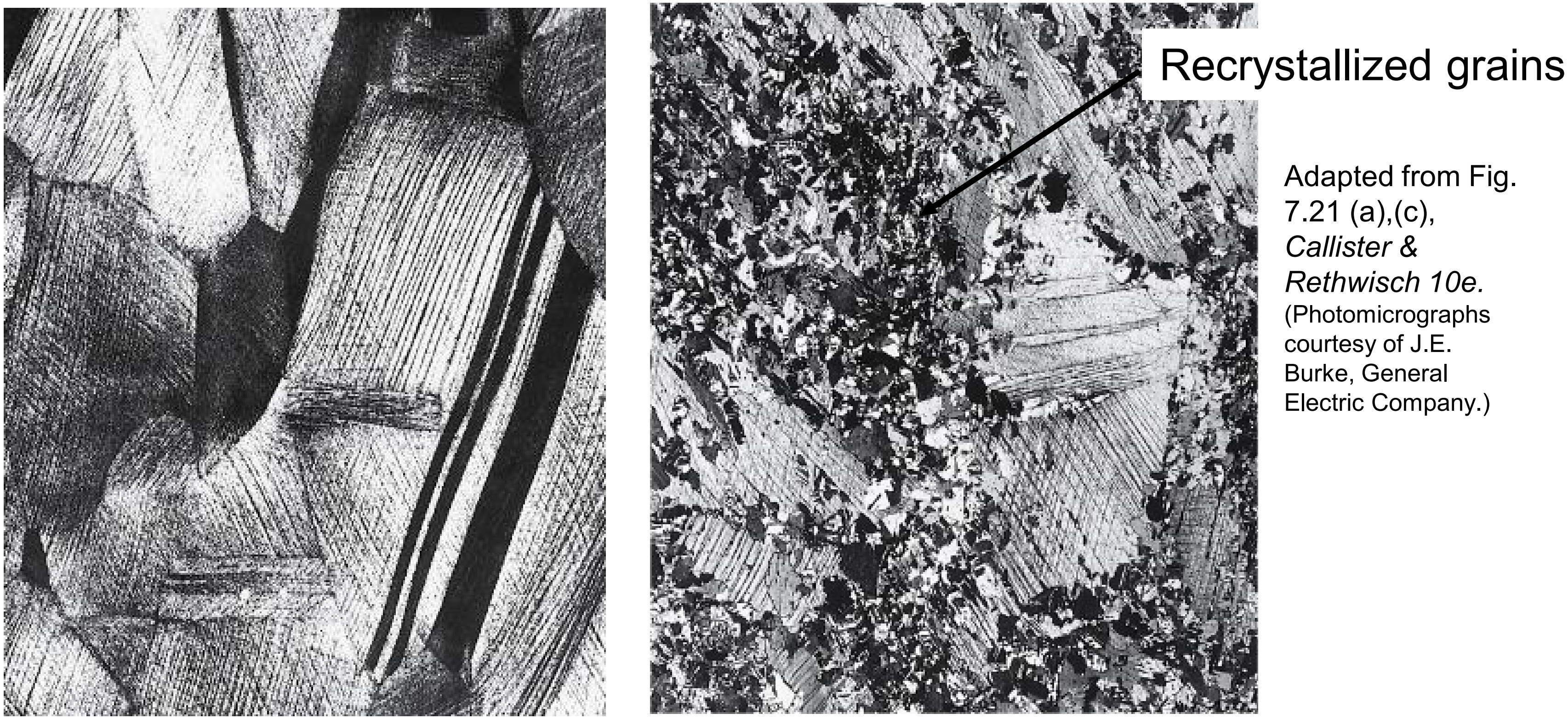


- Scenario 2



Recrystallization

- New grains form that:
  - -- have low dislocation densities
  - -- are small in size
  - -- consume and replace parent cold-worked grains.



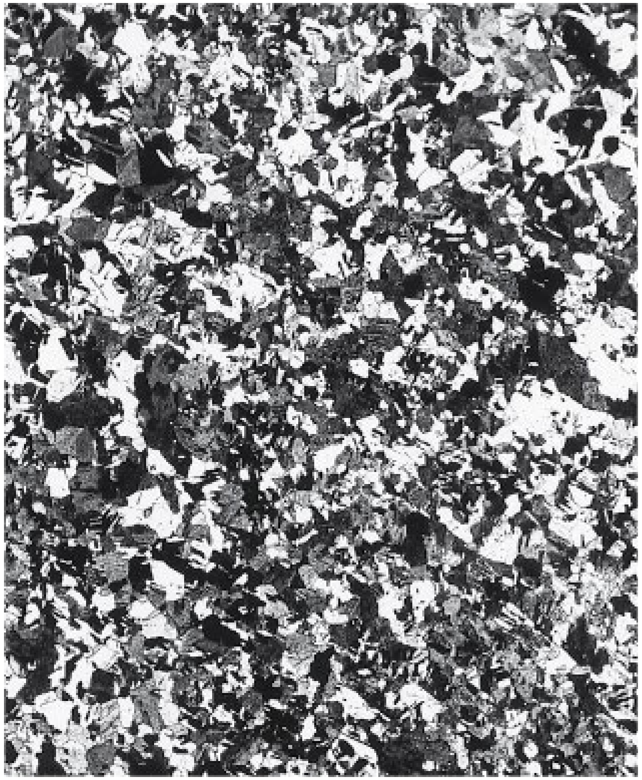
33%CW brass before heat treatment

After 4 sec. at 580° C

Adapted from Fig. 7.21 (a),(c), *Callister & Rethwisch 10e.* (Photomicrographs courtesy of J.E. Burke, General Electric Company.)

Recrystallization (cont.)

- All grains in cold-worked material have been consumed/replaced.



After 8 sec. at 580° C

Adapted from Fig. 7.21 (d), *Callister & Rethwisch 10e.* (Photomicrographs courtesy of J.E. Burke, General Electric Company.)

Recrystallization Temperature

T_R = recrystallization temperature = temperature at which recrystallization just reaches completion in 1 h.

$$0.3T_m < T_R < 0.6T_m$$

For a specific metal/alloy, T_R depends on:

- %CW -- T_R decreases with increasing %CW
- Purity of metal -- T_R decreases with increasing purity

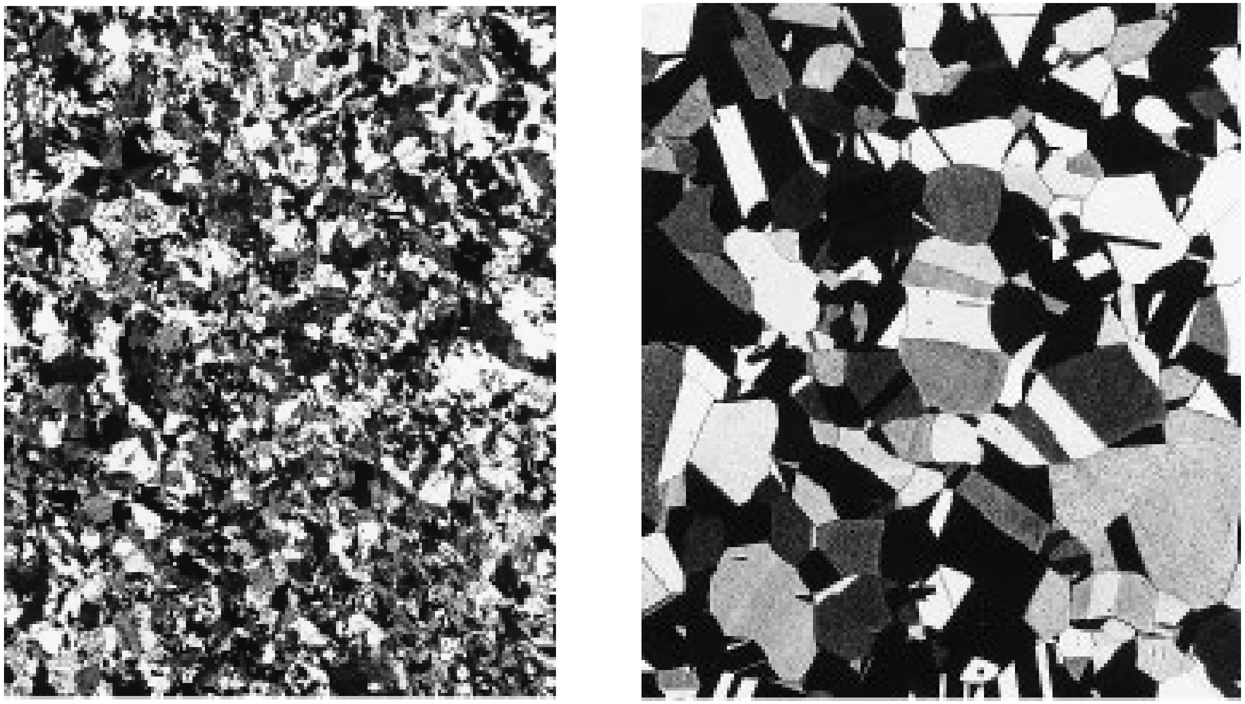
Cold Working vs. Hot Working

- Hot working → deformation above T_R

- Cold working → deformation below T_R

Grain Growth

- Grain growth occurs as heat treatment continues.
  - -- Average grain size increases
  - -- Small grains shrink (and ultimately disappear)
  - -- Large grains continue to grow



After 8 sec. at 580° C

After 15 min. at 580° C

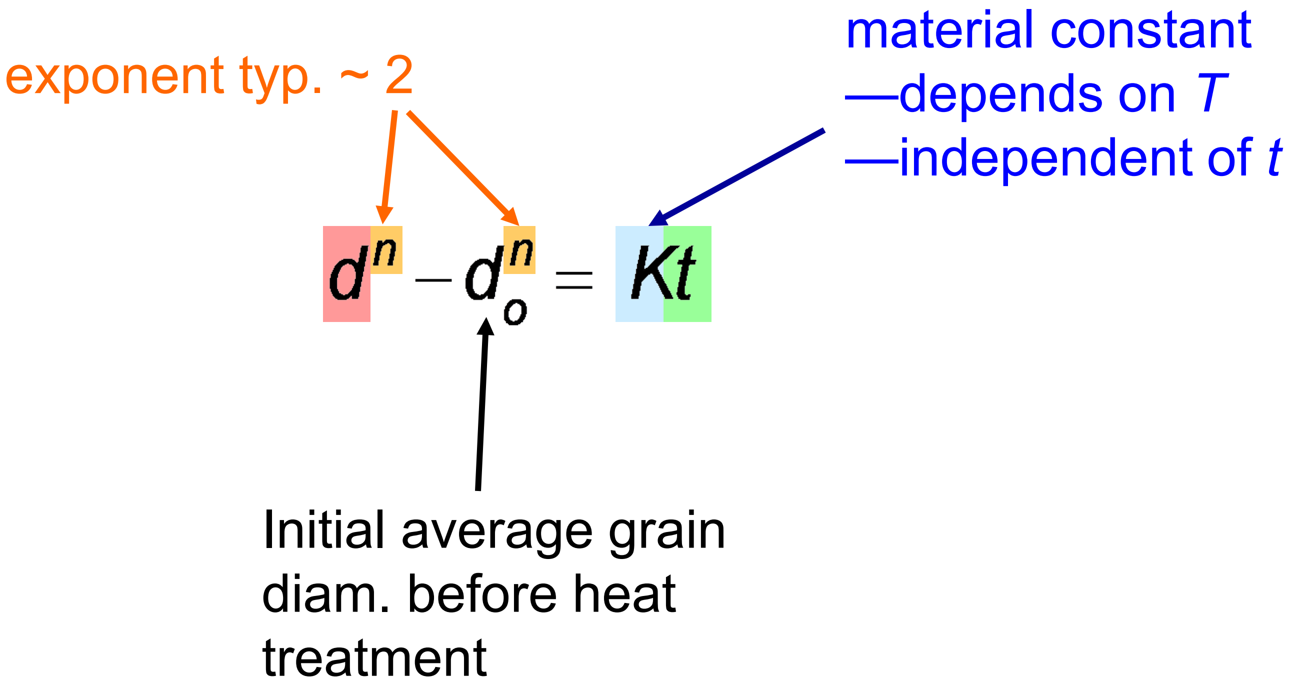
Adapted from Fig. 9.21 (d),(e), *Callister & Rethwisch 10e.* (Photomicrographs courtesy of J.E. Burke, General Electric Company.)

Grain Size Influences Properties

- Metals having small grains – relatively strong and tough at low temperatures
- Metals having large grains – good creep resistance at relatively high temperatures

Grain Growth (cont.)

- Empirical relationship—dependence of average grain size (*d*) on heat treating time (*t*):



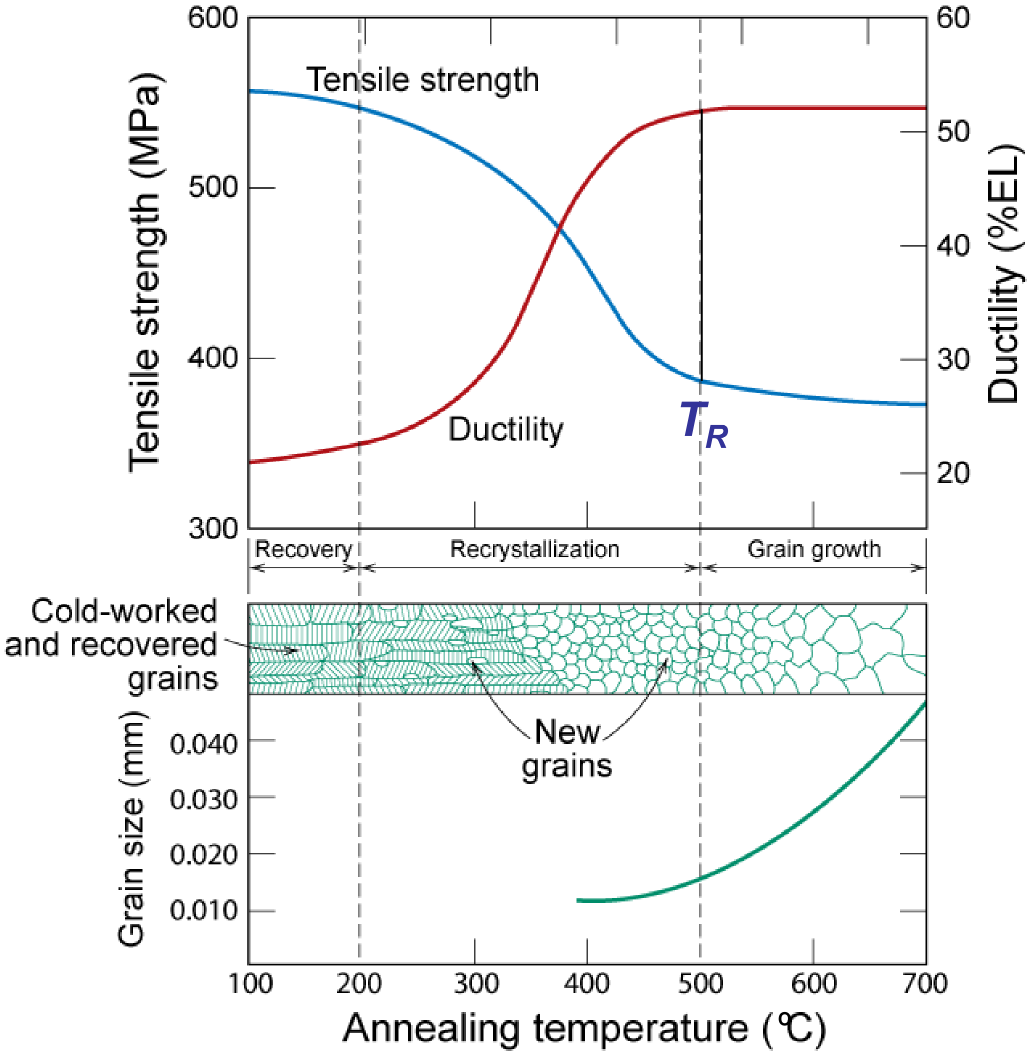
exponent typ. ~ 2

material constant
—depends on *T*
—independent of *t*

$$d^n - d_o^n = Kt$$

Initial average grain diam. before heat treatment

Recovery, Recrystallization, & Grain Growth Summary



T_R = recrystallization temperature

annealing time = 1 h

Fig. 7.22, *Callister & Rethwisch 10e.*
(Adapted from G. Sachs and K. R. Van Horn, *Practical Metallurgy, Applied Metallurgy and the Industrial Processing of Ferrous and Nonferrous Metals and Alloys*, 1940. Reproduced by permission of ASM International, Materials Park, OH.)

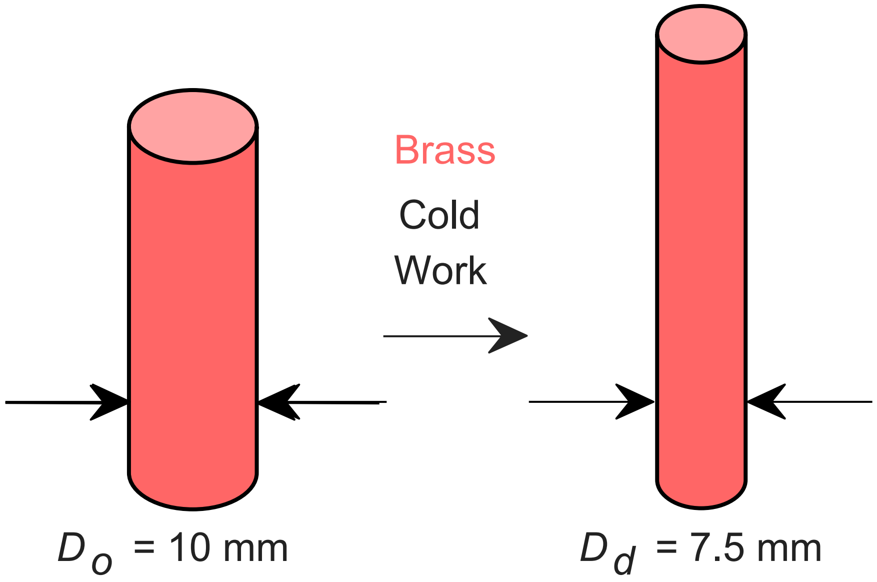
Design Problem - Description of Diameter Reduction Procedure

A cylindrical rod of brass originally 10 mm in diameter is to be cold worked by drawing. The circular cross section will be maintained during deformation. A cold-worked tensile strength in excess of 380 MPa and a ductility of at least 15 %EL are desired. Furthermore, the final diameter must be 7.5 mm. Explain how this may be accomplished.

Design Problem (cont.)

Solution:

First compute the %CW.



$$\%CW = \left(\frac{A_o - A_d}{A_o}\right) \text{ x } 100 = \left(1 - \frac{A_d}{A_o}\right) x\ 100$$

$$= \left[1 - \frac{\cancel{\pi}(D_d / \cancel{2})^2}{\cancel{\pi}(D_o / \cancel{2})^2}\right] \text{ x } 100 = \left[1 - \left(\frac{7.5 \text{ mm}}{10 \text{ mm}}\right)^2\right] \text{ x } 100 = 43.8\%$$

Design Problem Solution (cont.)

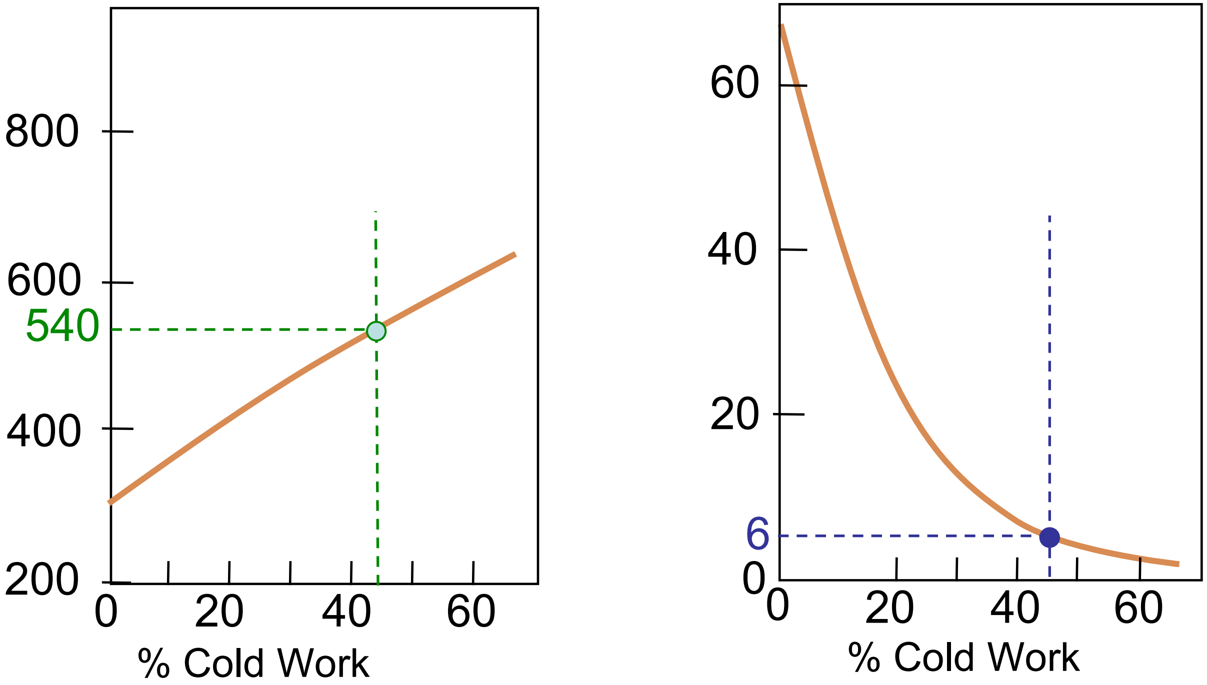


Fig. 7.19, *Callister & Rethwisch 10e.*

- For %CW = 43.8%
  - *TS* = 540 MPa   > 380 MPa
  - %*EL* = 6   < 15
- This doesn’t satisfy criteria… what other options are possible?

Design Problem Solution (cont.)

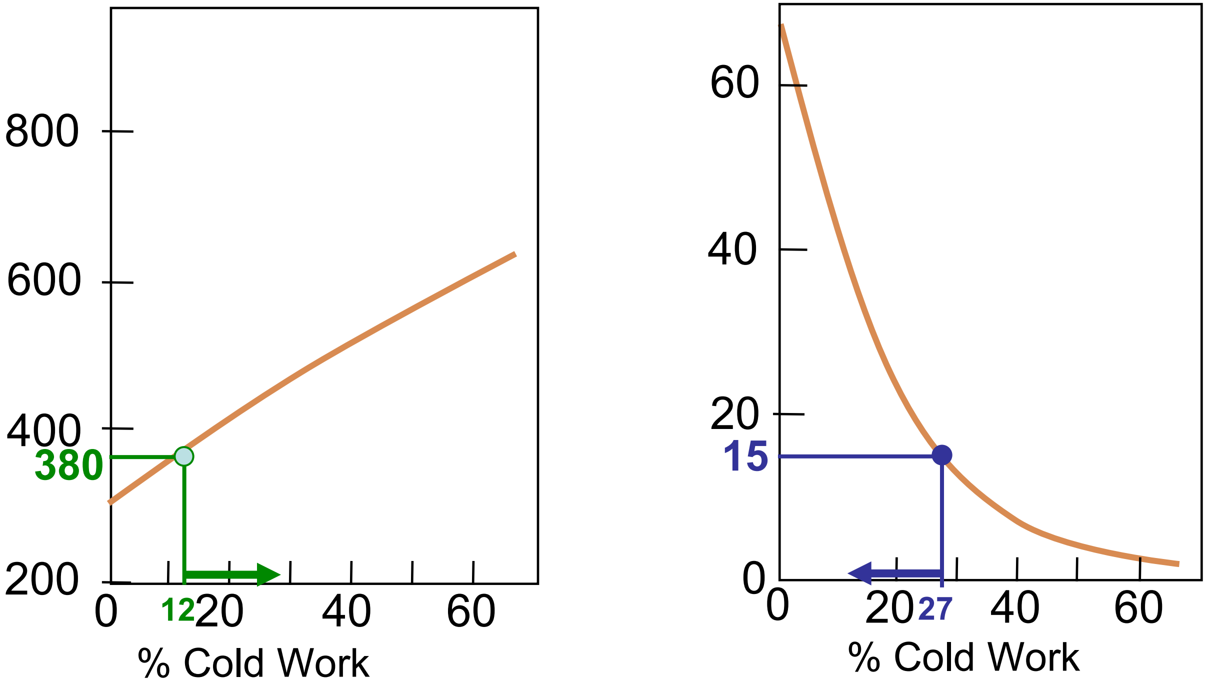


Fig. 7.19, *Callister & Rethwisch 10e.*

For *TS* > 380 MPa → > 12 %CW

For %*EL* > 15 → < 27 %CW

To meet criteria

—deformation requirement 12 < %CW < 27

Design Problem Solution (cont.)

Procedure: Cold work, anneal, then cold work again.

- To meet criteria, for 2nd deformation step: $12 < \%CW < 27$
  - We will deform to 20%CW
- Diameter after first cold work stage (but before 2nd cold work stage), D_i, calculated as follows:

$$\%CW = \left(1 - \frac{D_d^2}{D_i^2}\right) x\ 100 \quad \Rightarrow \quad 1 - \frac{D_d^2}{D_i^2} = \frac{\%CW}{100}$$

$$\frac{D_d}{D_i} = \left(1 - \frac{\%CW}{100}\right)^{0.5} \quad \Rightarrow \quad D_i = \frac{D_d}{\left(1 - \frac{\%CW}{100}\right)^{0.5}}$$

Intermediate diameter = $D_i = \frac{7.5\text{ mm}}{\left(1 - \frac{20\%CW}{100}\right)^{0.5}} = \boxed{8.39\text{ mm}}$

Design Problem Summary

Stage 1: Cold work – reduce diameter from 10 mm to 8.39 mm

$$\%CW_1 = \left[1 - \left(\frac{8.39\text{ mm}}{10\text{ mm}}\right)^2\right] x\ 100 = 29.6$$

Stage 2: Heat treat (allow recrystallization)

Stage 3: Cold work – reduce diameter from 8.39 mm to 7.5 mm

$$\%CW_2 = \left[1 - \left(\frac{7.5\text{ mm}}{8.39\text{ mm}}\right)^2\right] x\ 100 = 20$$

Therefore, all criteria satisfied

$$\%CW = 20$$
$$TS = 400\text{ MPa}$$
$$\%EL = 24$$

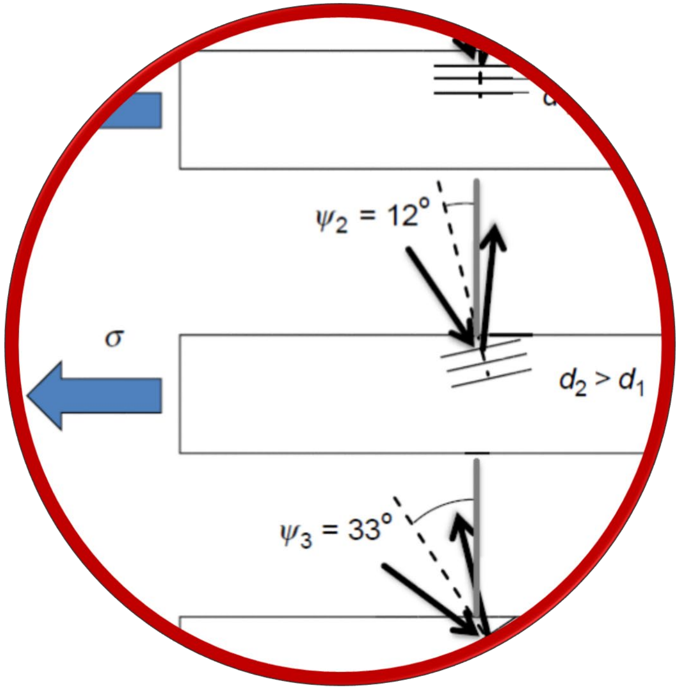
Summary

- Plastic deformation occurs by motion of dislocations
- Crystallographic considerations:
  - -- Minimum atomic distortion from dislocation motion
    - - in slip planes
    - - along slip directions
- Deformation of polycrystals—change of grain shapes
- Strength is increased by decreasing dislocation mobility.
- Strengthening techniques for metals:
  - -- grain size reduction
  - -- solid solution strengthening
  - -- strain hardening (cold working)

Summary (cont.)

- Heat treatment of deformed metal specimens:
  - -- Processes
    - - Recovery
    - - Recrystallization
    - - Grain growth
  - -- Consequences—property alterations
    - - Softer and weaker
    - - More ductile

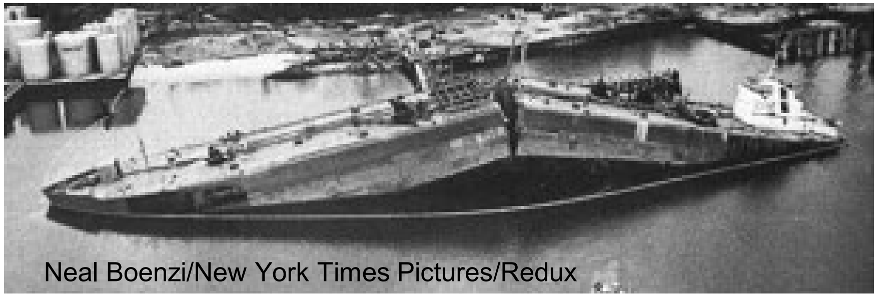
Fracture



Chapter 8: Failure

ISSUES TO ADDRESS...

- What are the different types of failure?
- Under what conditions/situation does each type occur?
- What is the mechanism associated with each failure type?
- What parameter is used to quantify a material's resistance to fracture?
- What measures may be taken to reduce the likelihood of each failure type?



Neal Boenzi/New York Times Pictures/Redux Pictures

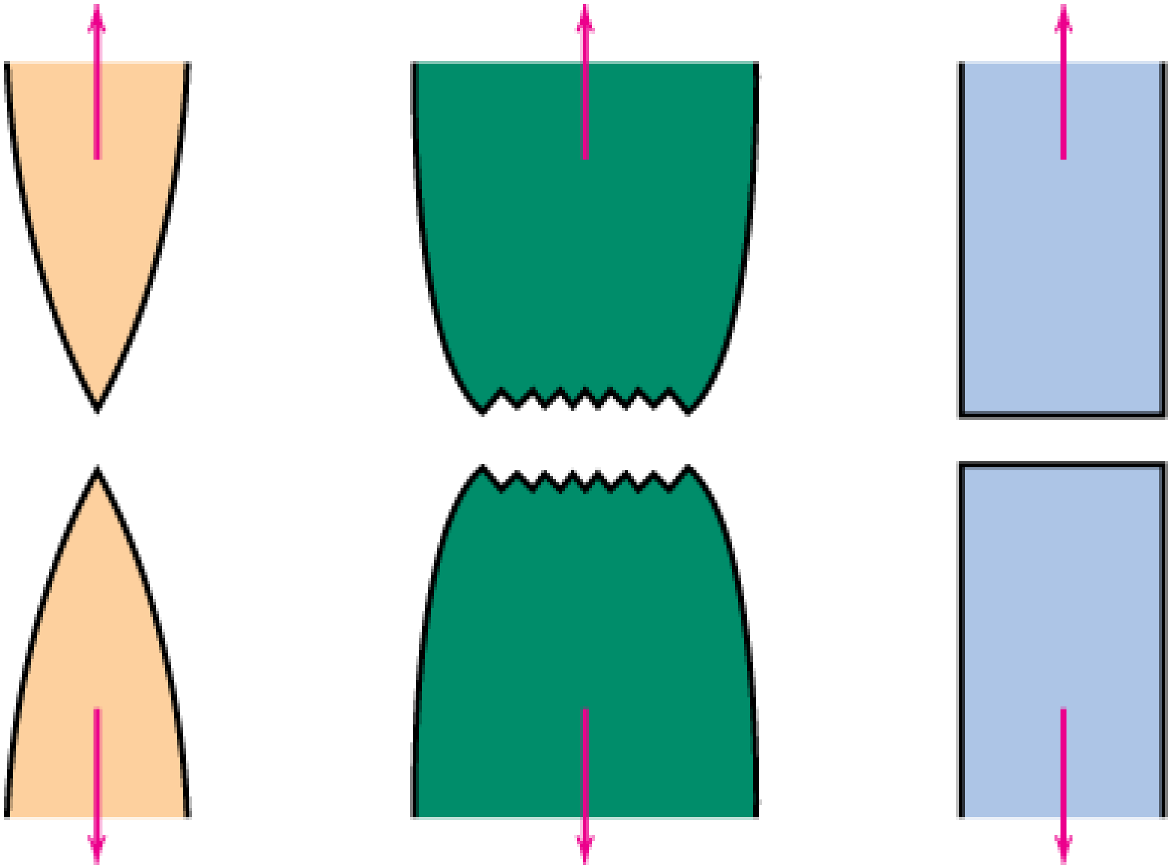
AP/Wide World Photos

Chapter-opening photographs, Chapter 8, *Callister & Rethwisch 10e.*

Fracture

- Simple fracture – the separation of a body into two or more pieces in response to a static stress
- Propagation of cracks accompanies fracture
- Two general types of fracture
  - Ductile
    - Slow crack propagation
    - Accompanied by significant plastic deformation
    - Fails with warning
  - Brittle
    - Rapid crack propagation
    - Little or no plastic deformation
    - Fails without warning
- Ductile fracture generally more desirable than brittle fracture

Fracture Profiles



Very Ductile

Moderately Ductile

Brittle

Adapted from Fig. 8.1, *Callister & Rethwisch 10e.*

Fracture Surface Photographs

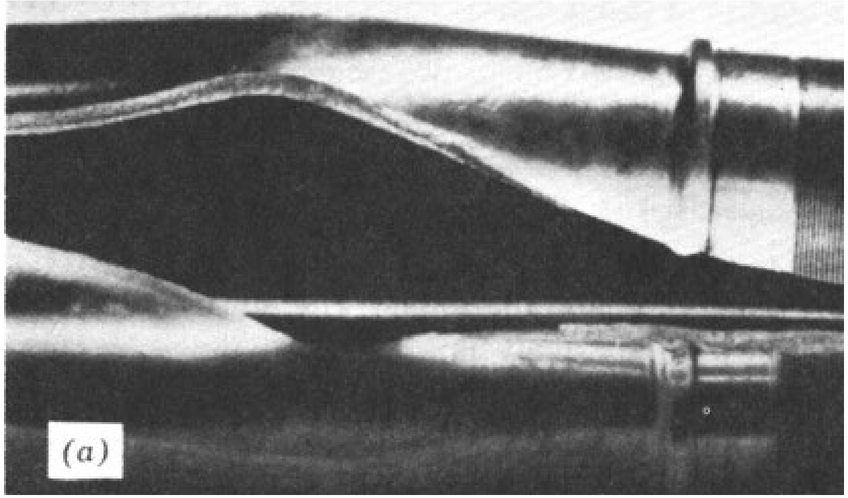
cup-and-cone fracture
- moderately ductile

brittle fracture
- totally brittle
- flat surfaces

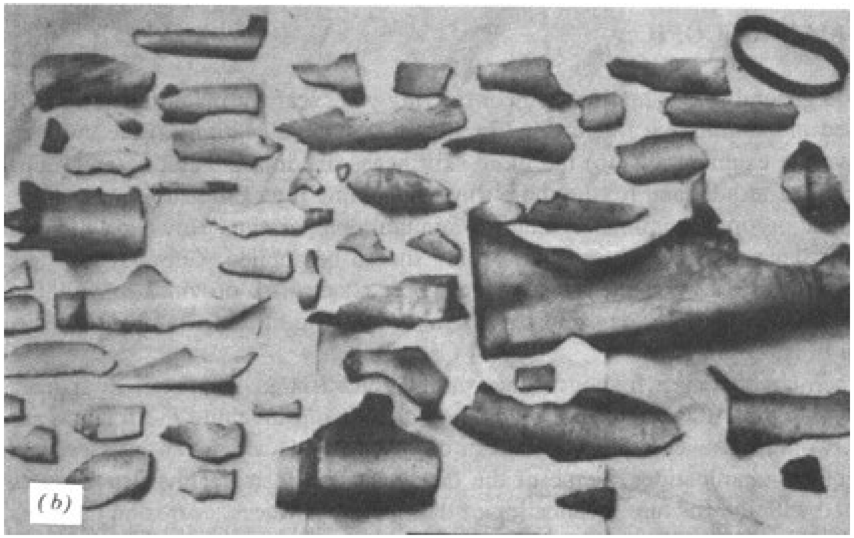
Fig. 8.3, *Callister & Rethwisch 10e.*

Examples of Ductile and Brittle Fracture of Pipes

- **Ductile fracture:**
  -- one piece
  -- large deformation



- **Brittle fracture:**
  -- many pieces
  -- small deformations



Figures from V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.), Fig. 4.1(a) and (b), p. 66 John Wiley and Sons, Inc., 1987. Used with permission.

Stages of Moderately Ductile Failure

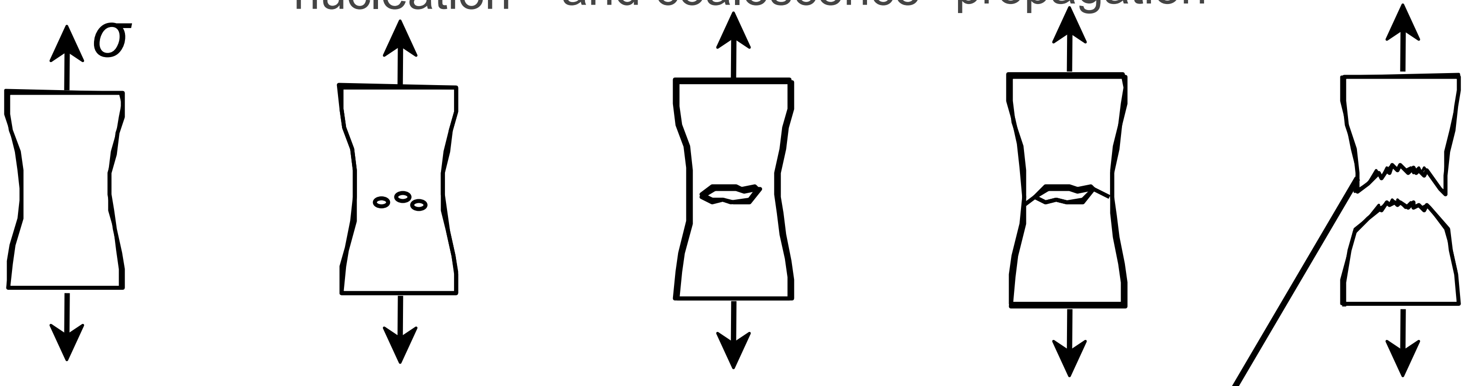
necking

void nucleation

void growth and coalescence

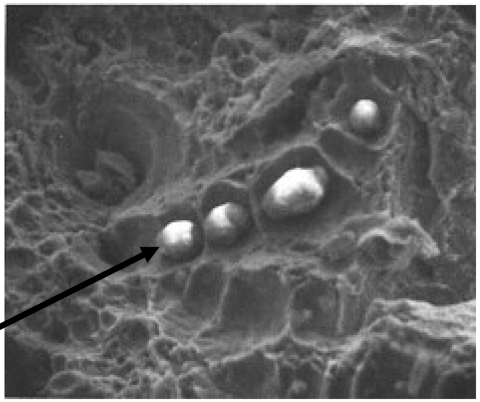
crack propagation

fracture

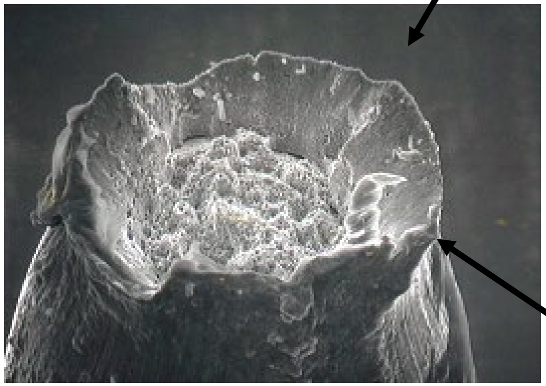


- Electron micrographs of fracture surfaces (steel)

particles serve as void nucleation sites.



From V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.), Fig. 11.28, p. 294, John Wiley and Sons, Inc., 1987. (Orig. source: P. Thornton, *J. Mater. Sci.*, Vol. 6, 1971, pp. 347-56.)



Fracture surface of tire cord wire loaded in tension. Courtesy of F. Roehrig, CC Technologies, Dublin, OH. Used with permission.

cup and cone fracture surface

Brittle Failure Surface Photographs

- Brittle fracture surface displays V-shaped, chevron markings
- V features point to the crack initiation site

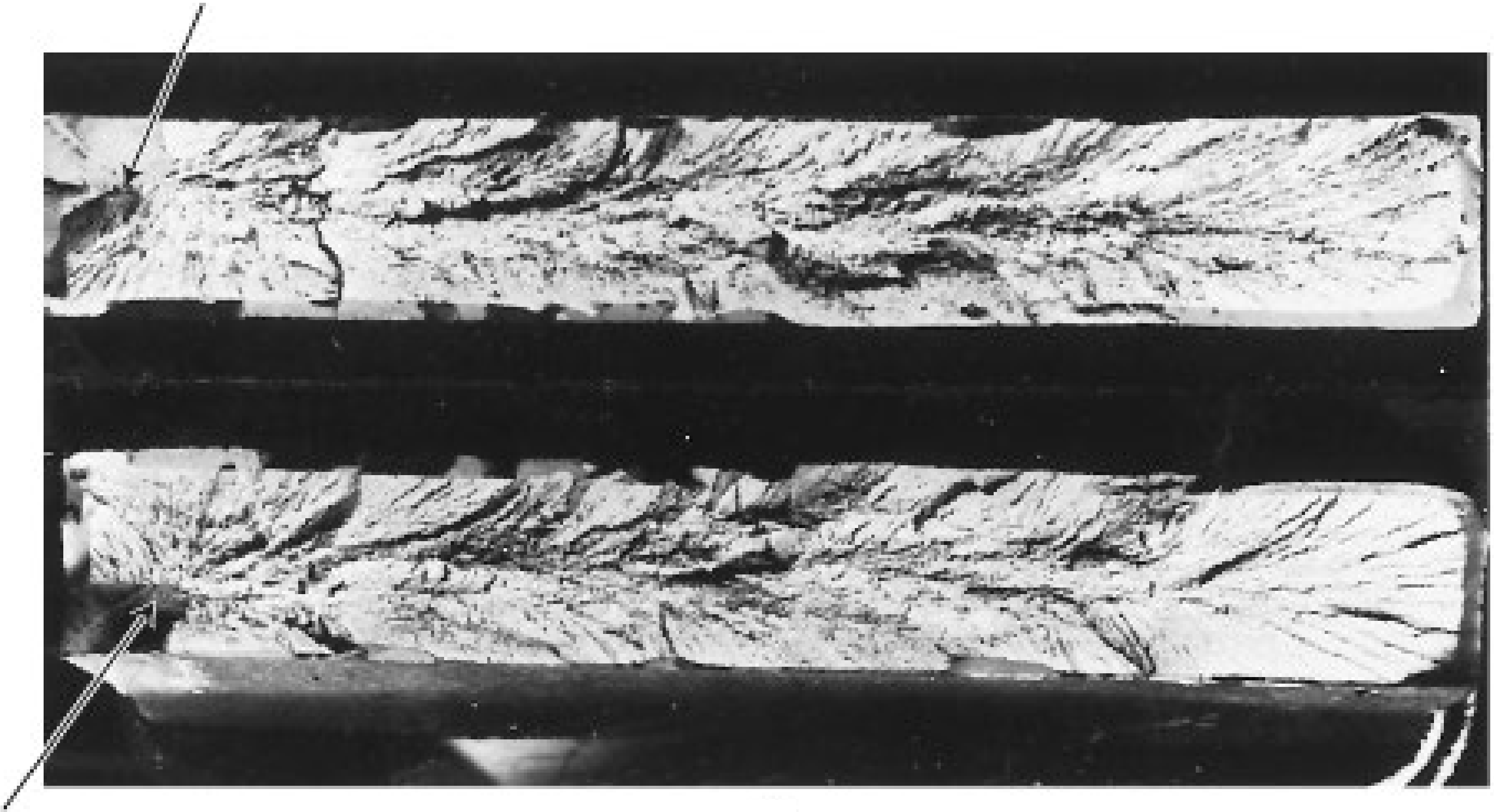
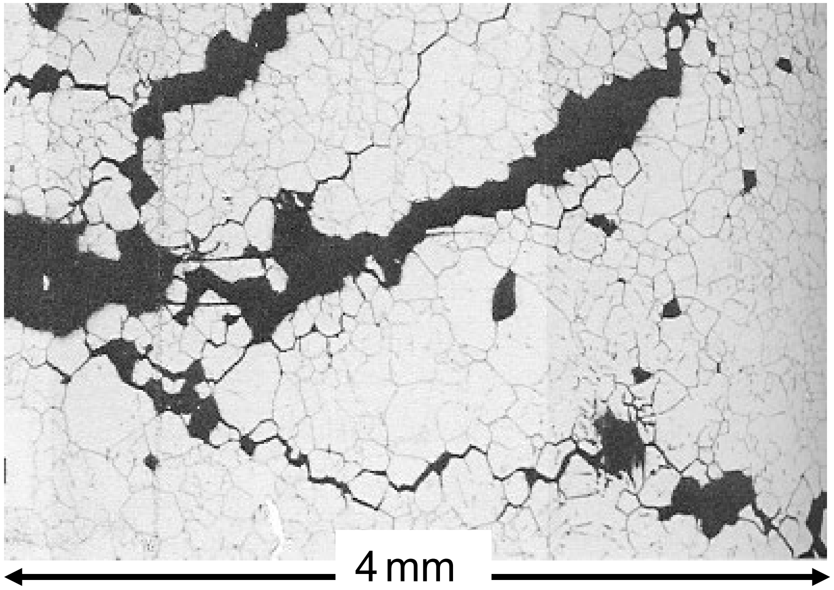


Fig. 8.5(a), *Callister & Rethwisch 10e.* [From R. W. Hertzberg, *Deformation and Fracture Mechanics of Engineering Materials*, 3rd edition. Copyright © 1989 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc. Photograph courtesy of Roger Slutter, Lehigh University.]

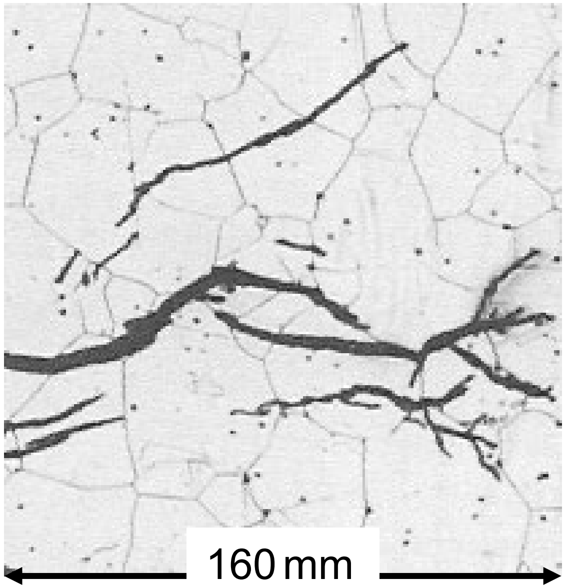
Photographs of Brittle Fracture Surfaces

- Intergranular crack propagation (between grains)

- Transgranular crack propagation (through grains)



304 S. Steel (metal)
Reprinted w/permission from "Metals Handbook", 9th ed, Fig. 633, p. 650. Copyright 1985, ASM International, Materials Park, OH. (Micrograph by J.R. Keiser and A.R. Olsen, Oak Ridge National Lab.)

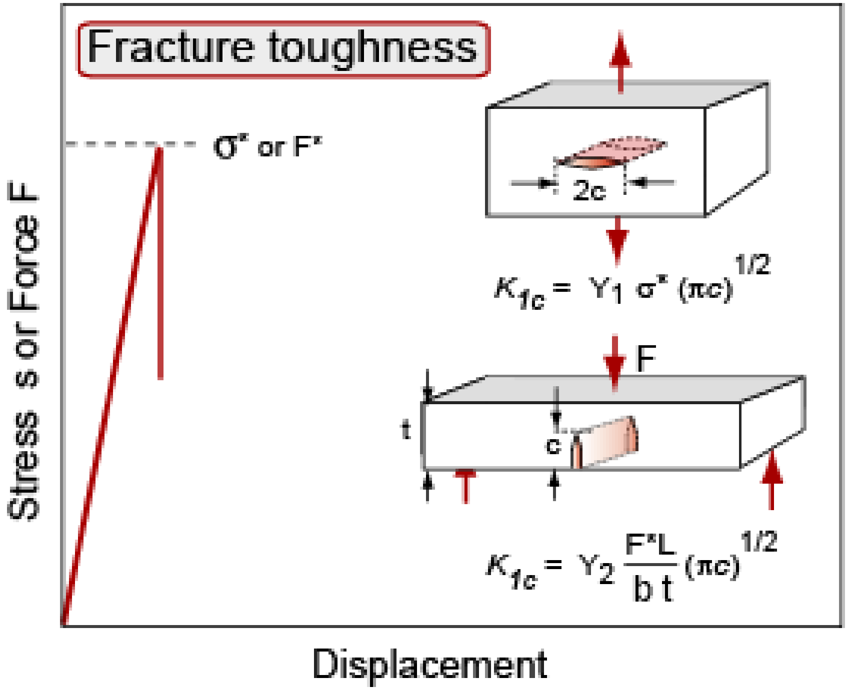


316 S. Steel (metal)
Reprinted w/ permission from "Metals Handbook", 9th ed, Fig. 650, p. 357. Copyright 1985, ASM International, Materials Park, OH. (Micrograph by D.R. Diercks, Argonne National Lab.)

Principles of Fracture Mechanics

- Fracture occurs as result of crack propagation
- Measured fracture strengths of most materials much lower than predicted by theory
  - microscopic flaws (cracks) always exist in materials
  - magnitude of applied tensile stress amplified at the tips of these cracks

Fracture mechanics



Definition:

Fracture toughness $$K_{1c} = Y_1 \sigma^* \sqrt{\pi c}$$

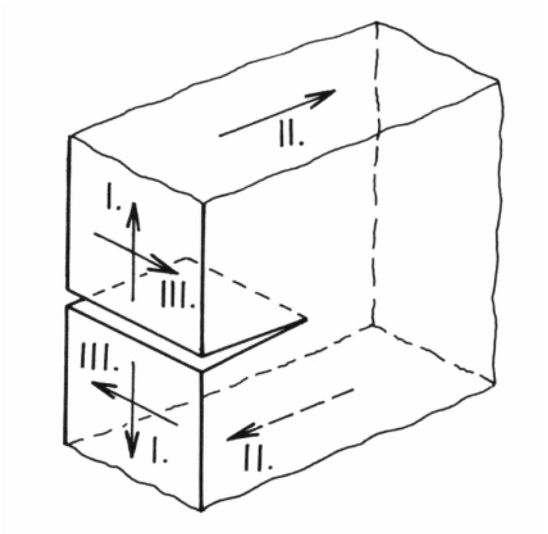
Fracture energy or toughness $$G = \frac{K_c^2}{E}$$

Range of fracture toughness:

0.3 MPa $m^{1/2}$ (glass) to 200 MPa $m^{1/2}$ GPa (steel)

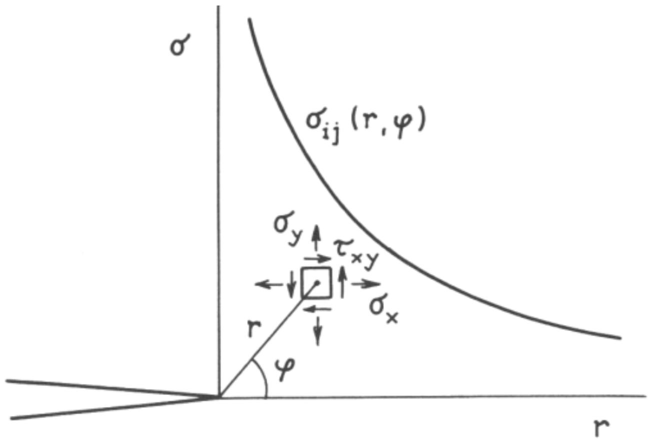
Empa
Materials Science and Technology

Stress distribution around the crack tip



Loading modes:

- I: tensile
- II: in-plane shear
- III: out of plane shear



The Sneddon equation describes the stress distribution around a crack tip:

$$\sigma_{n,ij}(r,\varphi) = \frac{K_n}{\sqrt{2\pi r}} f_{n,ij}(\varphi)$$

Assumption: r is smaller than the crack length a, but larger than characteristic microstructual length scale.

Stress intensity factors

The stress intensity factors are given by:

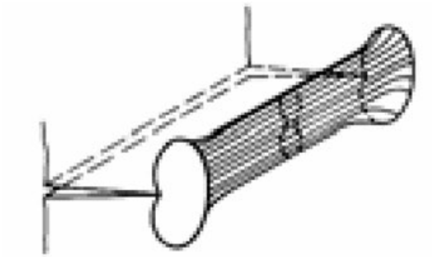
$$K_n = \sigma Y \sqrt{a}$$ with

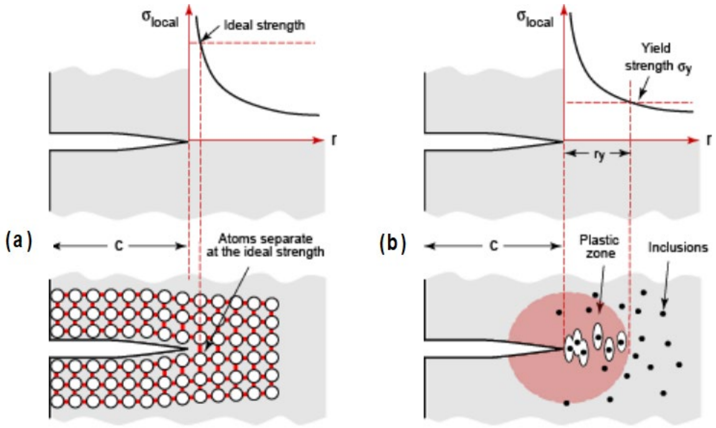
Y geometry factor
a crack length
σ stress

K_n depends on

- Shape and size of the sample and the crack
- Model and magnitude of the loading

Especially for ductile materials, a plastic zone develops at the crack tip:





$$r_y = \frac{K_C^{\;2}}{2\pi\sigma_f^{\;2}}$$

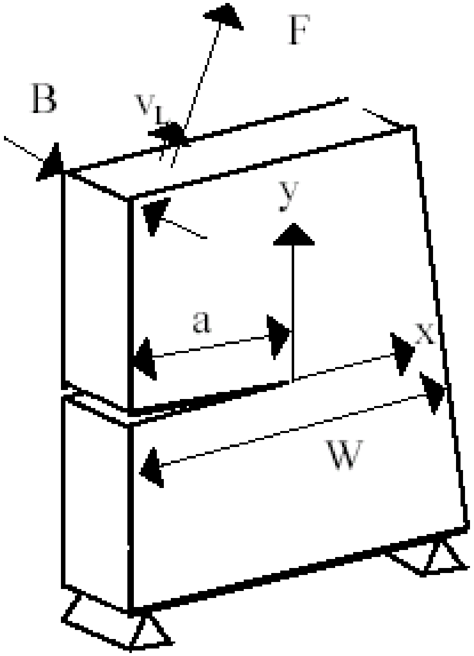
Critical energy release rate

Energy release rate for a mode I crack

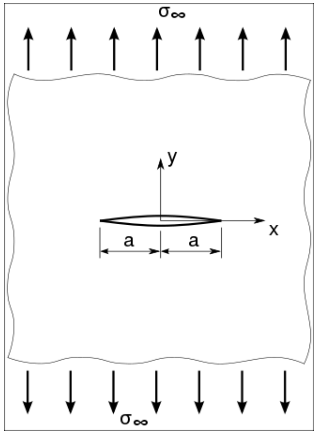
$$G = -\frac{1}{B}\frac{dW_{el}}{da}$$

Critical energy release rate:

$$G_c = 2(\gamma_s + \gamma_p)$$



Crack propagation criteria



Griffith criterion

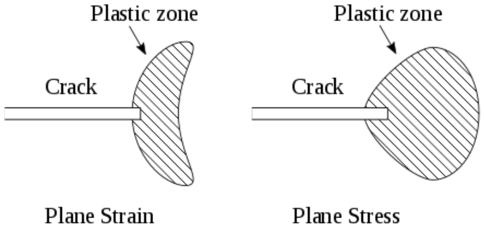
$G > G_c$

Irwin criteriom

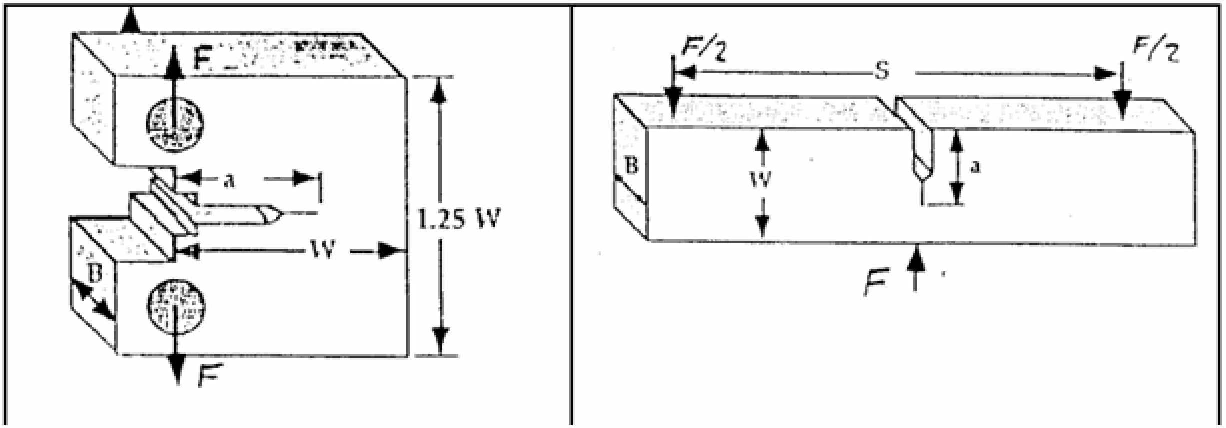
$K > K_c$

For linear elastic fracture mechanics, there is a relationship between these criteria:

$$K_{Ic} = \sqrt{\frac{E}{1-\nu^2} G_{Ic}}$$

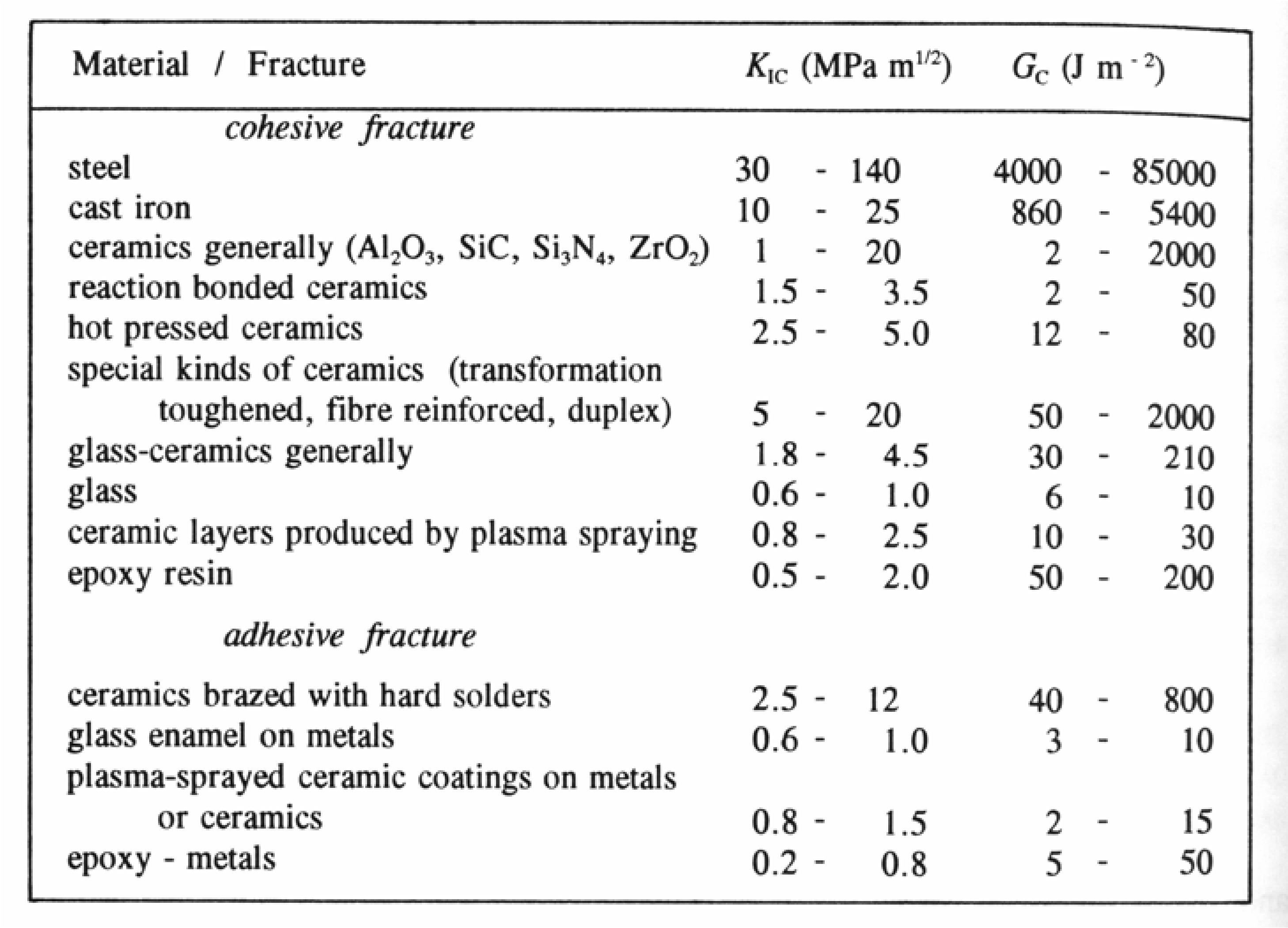


Measuring fracture toughness



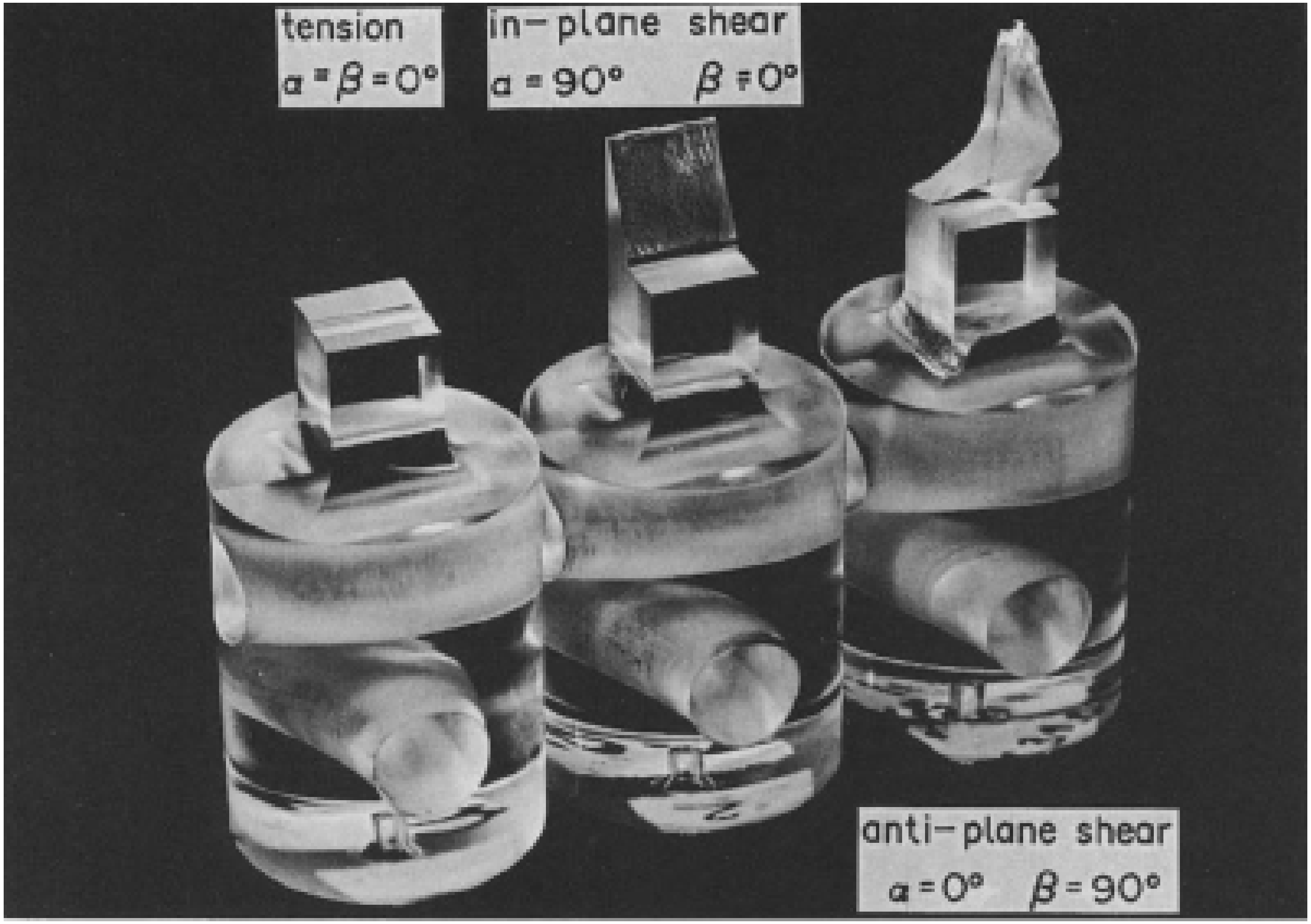
	Compact tension sample	3 point bending sample
K_I	$K_I = \frac{F}{B \cdot (W-a)^{3/2}} \cdot (1.46a + 2.51W)$	$K_I = \frac{0.92 \cdot F \cdot S}{B \cdot (W-a)^{1.5}}$

Fracture toughness – typical values



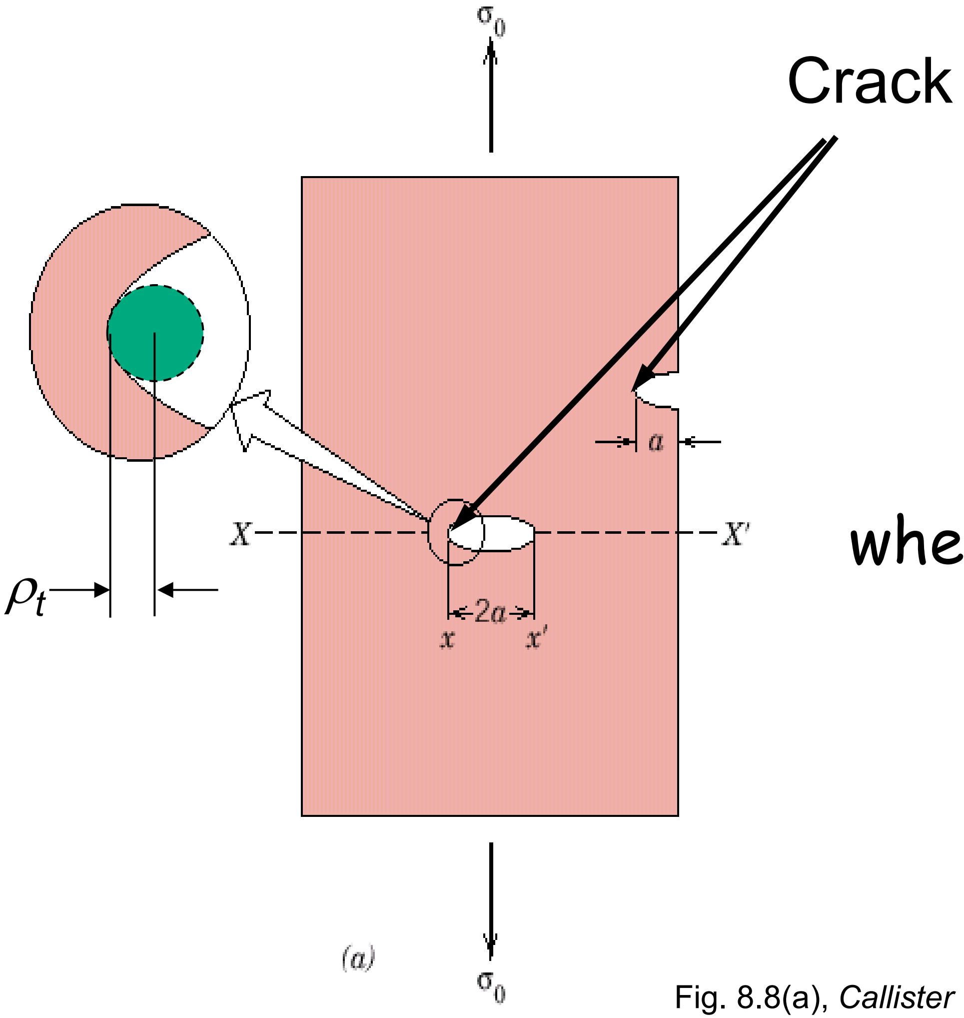
Material / Fracture	K_{IC} (MPa $m^{1/2}$)	G_C (J m^{-2})
cohesive fracture		
steel	30 - 140	4000 - 85000
cast iron	10 - 25	860 - 5400
ceramics generally (Al_2O_3, SiC, Si_3N_4, ZrO_2)	1 - 20	2 - 2000
reaction bonded ceramics	1.5 - 3.5	2 - 50
hot pressed ceramics	2.5 - 5.0	12 - 80
special kinds of ceramics (transformation toughened, fibre reinforced, duplex)	5 - 20	50 - 2000
glass-ceramics generally	1.8 - 4.5	30 - 210
glass	0.6 - 1.0	6 - 10
ceramic layers produced by plasma spraying	0.8 - 2.5	10 - 30
epoxy resin	0.5 - 2.0	50 - 200
adhesive fracture		
ceramics brazed with hard solders	2.5 - 12	40 - 800
glass enamel on metals	0.6 - 1.0	3 - 10
plasma-sprayed ceramic coatings on metals or ceramics	0.8 - 1.5	2 - 15
epoxy - metals	0.2 - 0.8	5 - 50

Mode-dependent fracture



Fracture Mechanics (cont.)

Flaws are Stress Concentrators!



$$\sigma_m = 2\sigma_o \left(\frac{a}{\rho_t}\right)^{1/2}$$

where

ρ_t = radius of curvature

σ_o = applied stress

σ_m = stress at crack tip

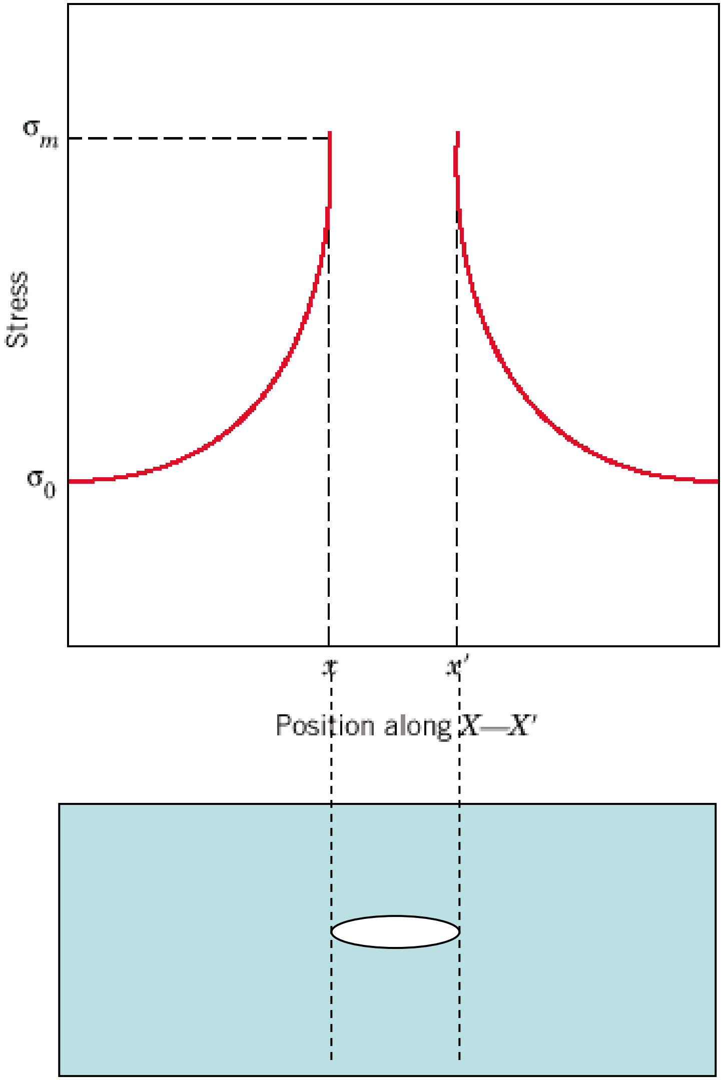
Fig. 8.8(a), *Callister & Rethwisch 10e.*

Fracture Mechanics (cont.)

Stress Concentration at Crack Tip

K_t = stress concentration factor

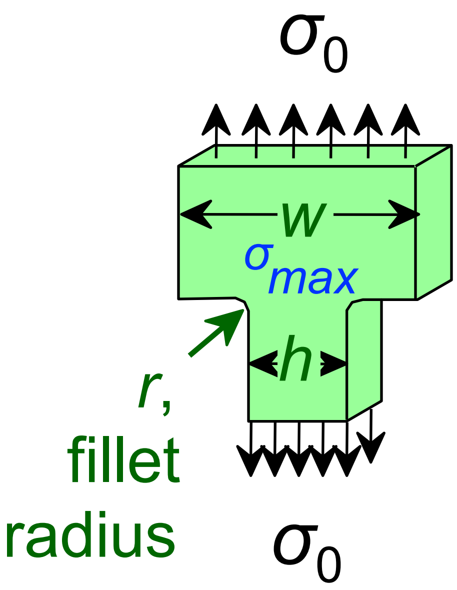
$$K_t = \frac{\sigma_m}{\sigma_o}$$

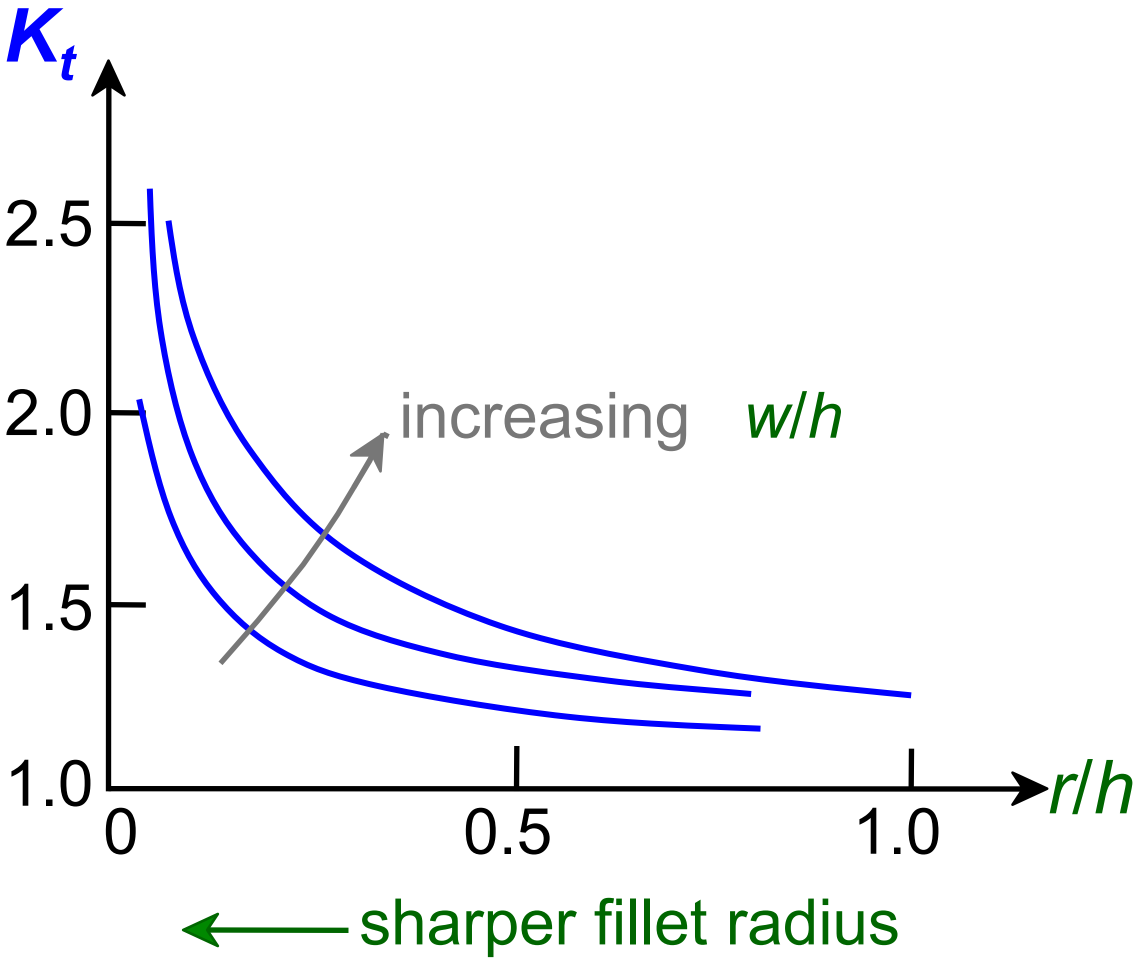


Adapted from Fig. 8.8(b), *Callister & Rethwisch 10e.*

Fracture Mechanics (cont.)

- **Avoid sharp corners!**

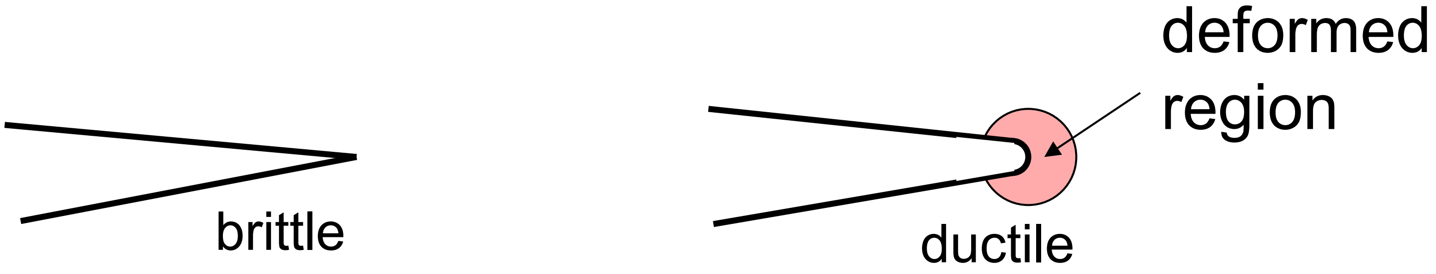




Adapted from Fig. 8.2W(c), *Callister 6e.* (Fig. 8.2W(c) is from G.H. Neugebauer, *Prod. Eng.* (NY), Vol. 14, pp. 82-87 1943.)

Crack Propagation

- Stress concentration higher for sharp cracks—propagate at lower stresses than cracks with blunt tips
- For ductile materials—plastic deformation at crack tip when stress reaches yield strength—tip blunted—lowers stress conc.



Criterion for Crack Propagation

Critical stress for crack propagation (σ_c) of brittle materials

$$\sigma_c = \left(\frac{2E\gamma_s}{\pi a} \right)^{1/2}$$

where

- σ_c = crack-tip stress
- E = modulus of elasticity
- γ_s = specific surface energy
- a = one half length of internal crack

For ductile materials replace γ_s with $\gamma_s + \gamma_p$ where γ_p is plastic deformation energy

- materials have numerous cracks with different lengths and orientations
- crack propagation (and fracture) occurs when $\sigma_m > \sigma_c$ for crack with lowest σ_c
- Largest, most highly stressed cracks grow first!

Fracture Toughness

- Measure of material's resistance to brittle fracture when a crack is present
- Defined as

$$K_c = Y\sigma_c\sqrt{\pi a}$$

K_c = fracture toughness [MPa $\sqrt{m}$]

Y = dimensionless parameter

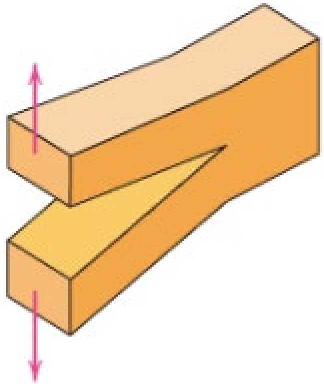
σ_c = critical stress for crack propagation [MPa]

a = crack length [m]

- For planar specimens with cracks much shorter than specimen width, $Y \approx 1$

Plane Strain Fracture Toughness

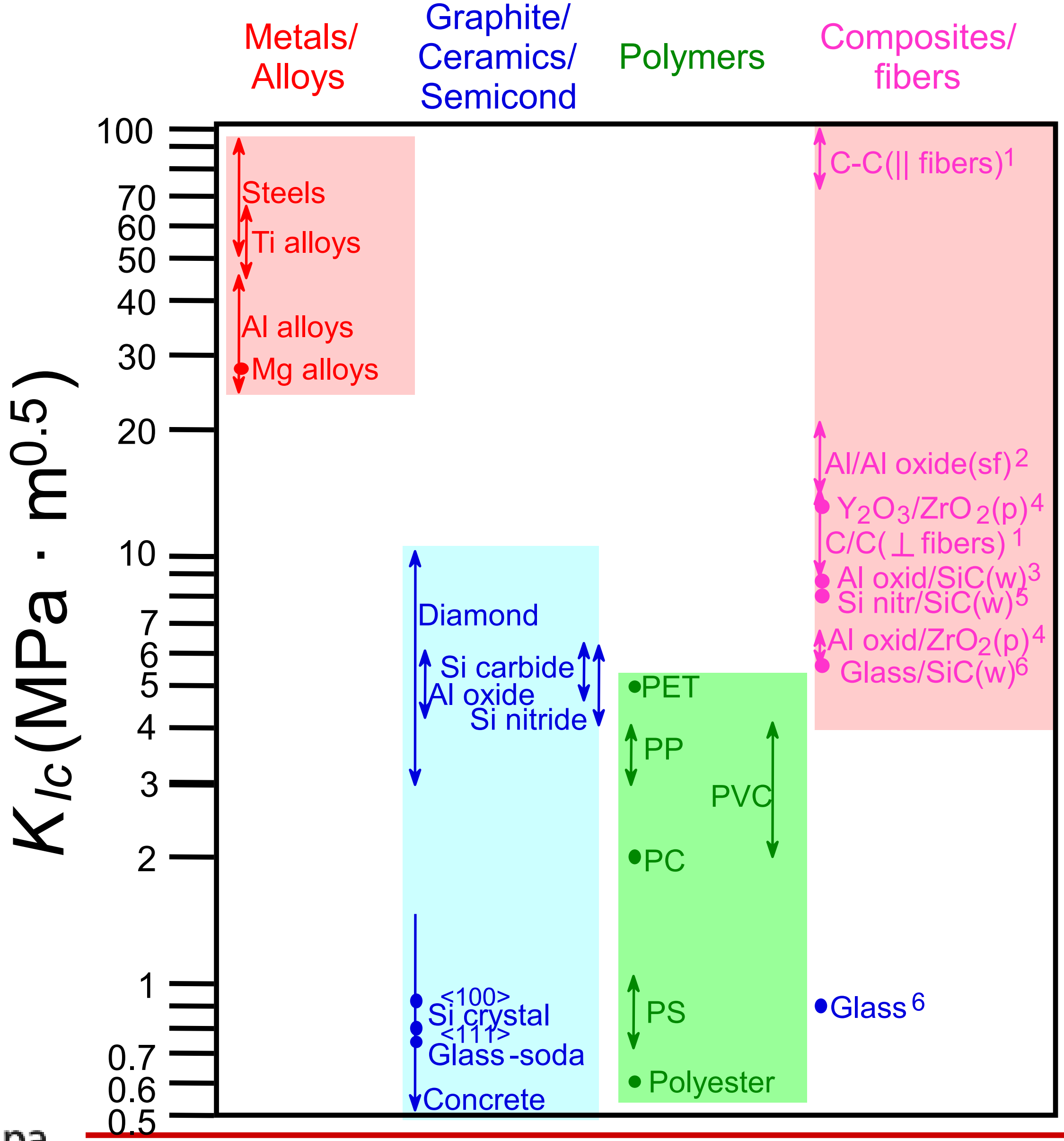
- For specimen thickness much greater than crack dimension, K_C independent of thickness
  - Condition of plane strain exists
  - Leads to plane strain fracture toughness, K_{ic}, where I indicates mode I crack displacement



Mode I, opening or tensile mode of crack surface displacement

- values of K_{ic} relatively high for ductile materials and low for brittle ones

Fracture Toughness Ranges



Based on data in Table B.5, *Callister & Rethwisch 10e.*

Composite reinforcement geometry is: f = fibers; sf = short fibers; w = whiskers; p = particles. Addition data as noted (vol. fraction of reinforcement):

1. (55vol%) *ASM Handbook*, Vol. 21, ASM Int., Materials Park, OH (2001) p. 606.
2. (55 vol%) Courtesy J. Cornie, MMC, Inc., Waltham, MA.
3. (30 vol%) P.F. Becher et al., *Fracture Mechanics of Ceramics*, Vol. 7, Plenum Press (1986). pp. 61-73.
4. Courtesy CoorsTek, Golden, CO.
5. (30 vol%) S.T. Buljan et al., "Development of Ceramic Matrix Composites for Application in Technology for Advanced Engines Program", ORNL/Sub/85-22011/2, ORNL, 1992.
6. (20vol%) F.D. Gace et al., *Ceram. Eng. Sci. Proc.*, Vol. 7 (1986) pp. 978-82.

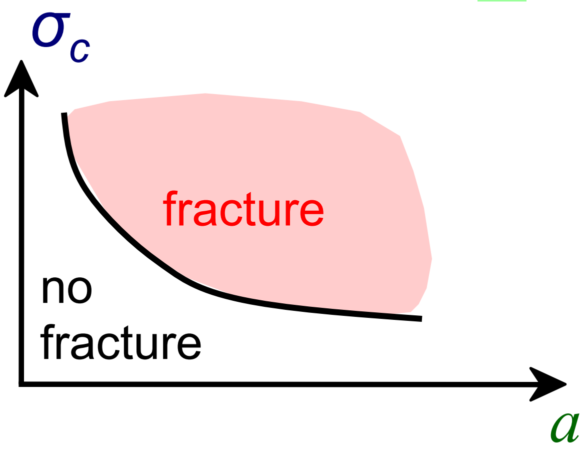
Design Against Fracture

- Crack growth condition:

$$K_{Ic} < Y\sigma_c\sqrt{\pi a}$$

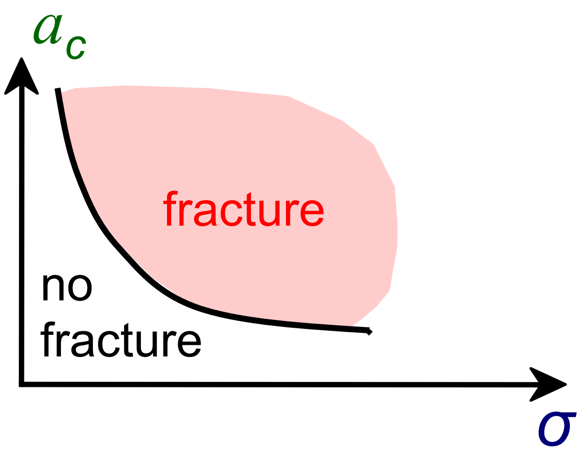
--Scenario 1: K_{Ic} and flaw size a specified - dictates max. design (critical) stress.

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}}$$



--Scenario 2: K_{ic} and stress level specified - dictates max. allowable flaw size.

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2$$



Design Example: Aircraft Wing

An aircraft component is made from a material has K_{Ic} = 26 MPa-m$^{0.5}$ It has been determined that fracture results at a stress of 112 MPa when the maximum (critical) internal crack length is 9.0 mm. For this same component and alloy, compute the stress level at which fracture will occur for a critical internal crack length of 4.0 mm.

Design Example: Aircraft Wing (cont.)

Solution: Given that fracture occurs for same component using same alloy the parameter *Y* will be the same for both situations. Solving for *Y* for the conditions under which fracture occurred using Equation 8.5.

$$Y = \frac{K_{Ic}}{\sigma_c \sqrt{\pi a}} = \frac{26 \text{ MPa-m}^{0.5}}{(112 \text{ MPa})\sqrt{\pi (9.0 \text{ mm})\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)}} = 1.38$$

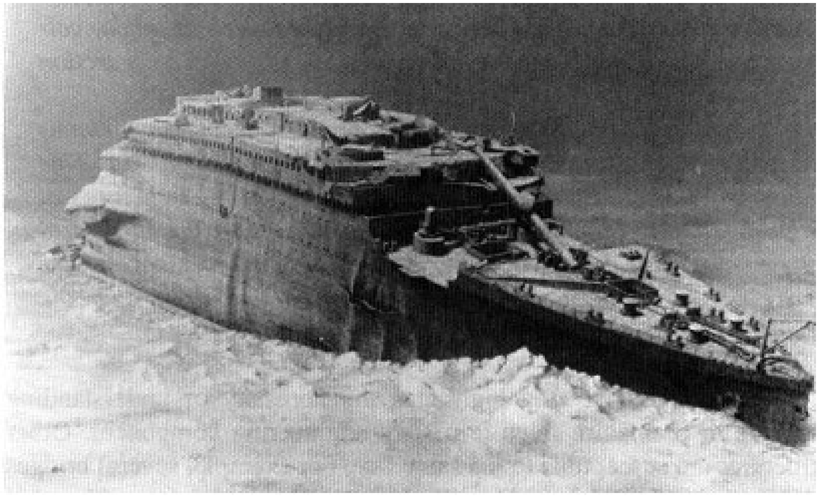
Now we will solve for σ_c using Equation 8.6 as

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{26 \text{ MPa-m}^{0.5}}{(1.38)\sqrt{\pi (4.0 \text{ mm})\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)}}$$

Answer: $\sigma_c = 168 \text{ MPa}$

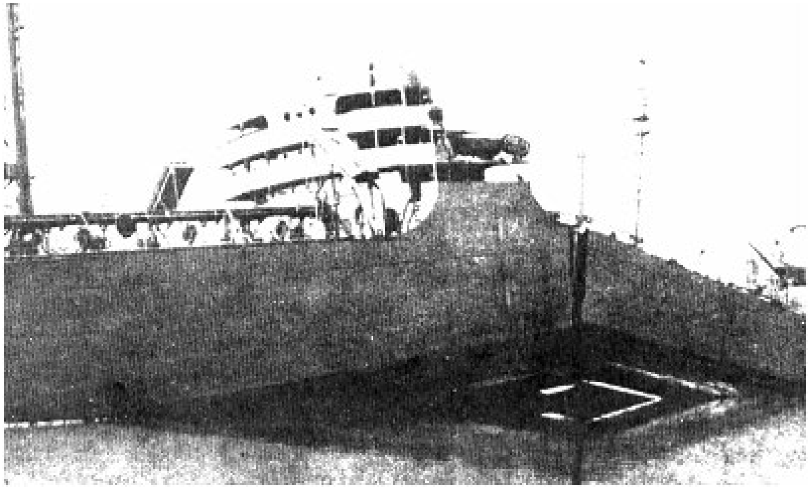
Brittle Fracture of Ductile Materials

- **Pre-WWII: The Titanic**



Reprinted w/ permission from R.W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials", (4th ed.) Fig. 7.1(a), p. 262, John Wiley and Sons, Inc., 1996. (Orig. source: Dr. Robert D. Ballard, *The Discovery of the Titanic*.)

- **WWII: Liberty ships**



Reprinted w/ permission from R.W. Hertzberg, "Deformation and Fracture Mechanics of Engineering Materials", (4th ed.) Fig. 7.1(b), p. 262, John Wiley and Sons, Inc., 1996. (Orig. source: Earl R. Parker, "Behavior of Engineering Structures", Nat. Acad. Sci., Nat. Res. Council, John Wiley and Sons, Inc., NY, 1957.)

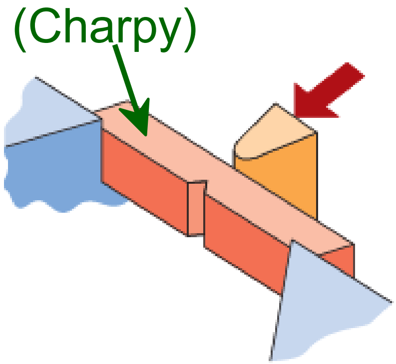
- Ships failed in a brittle manner though constructed of steel that, from tension tests, is normally ductile

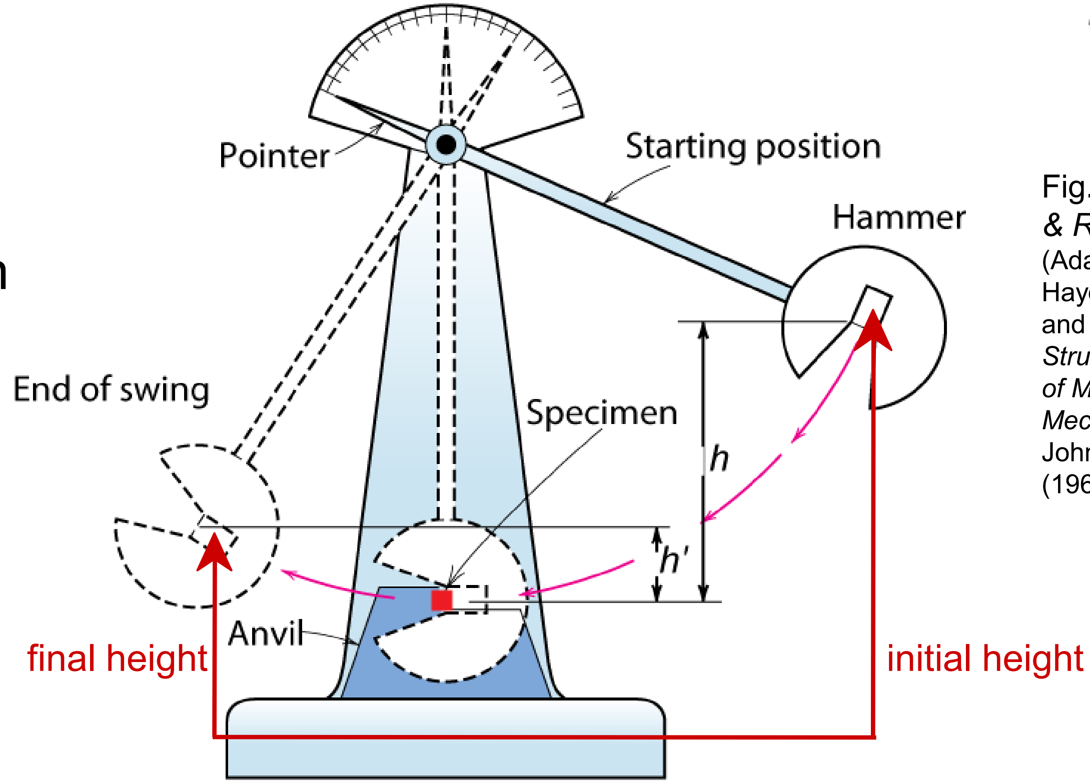
Testing Ductile Materials for Brittle Failure

Impact Test

- Test conditions promoting brittle fracture:
  - -- High strain rate
  - -- Deformation at low temperatures
  - -- Presence of a notch

(Charpy)



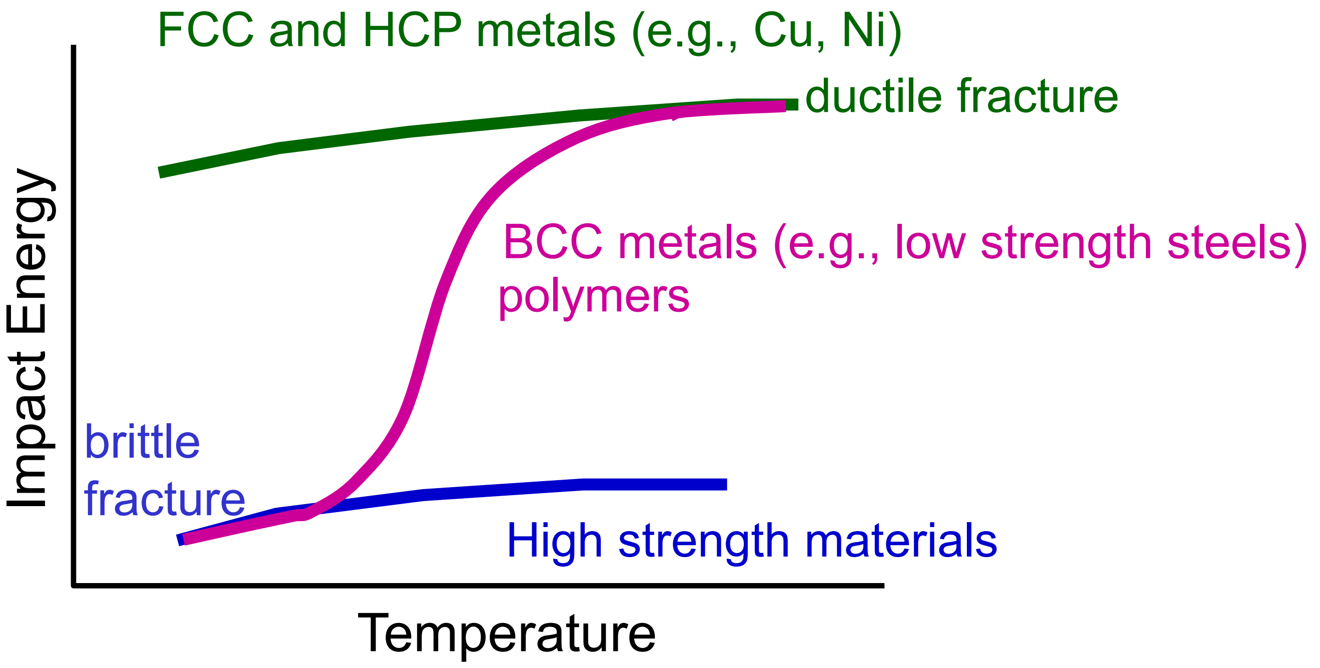


Impact energy computed from difference between initial height *h* and final height *h*'

Fig. 8.13(b), *Callister & Rethwisch 10e.* (Adapted from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, John Wiley and Sons, Inc. (1965) p. 13.)

Influence of *T* on Impact Energy

- When impact tests conducted as function of temperature—three kinds of behavior observed for metals
- Some BCC metals exhibit Ductile-to-Brittle Transition Temperature (DBTT)



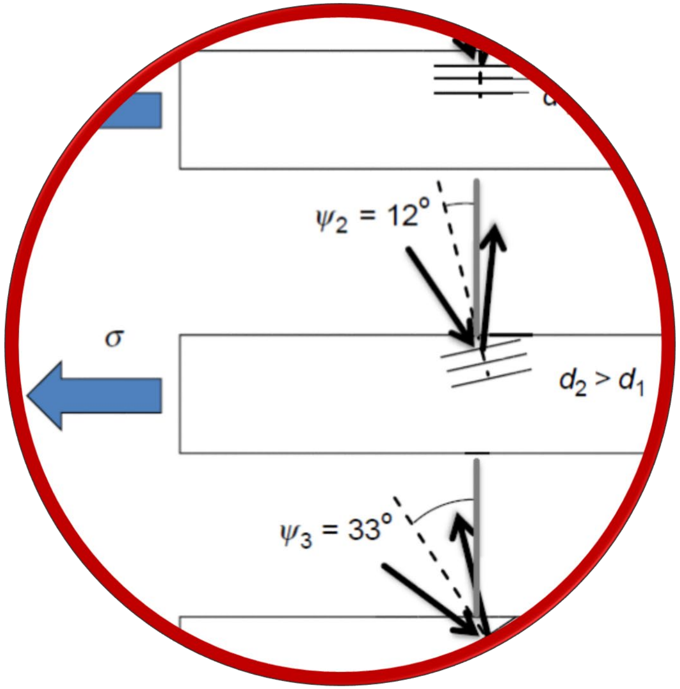
Adapted from Fig. 8.16, *Callister & Rethwisch 10e.*

Metals having DBTT should only be used at temperatures where ductile.

SUMMARY

- Simple fracture – one type of failure
  - Occurs by crack propagation
  - Ductile fracture: some plastic deformation – slow crack propagation
  - Brittle fracture: no plastic deformation – crack propagation
  - Fracture surfaces – different for ductile and brittle
- Small cracks or flaws exist in all materials
  - Applied tensile stress amplified at tips of flaws
  - Fracture – when stress at tip of crack reaches theoretical strength
- Fracture toughness – measurement of material's resistance to brittle fracture
  - A function of applied stress and crack length
- Impact tests – Impact energy measured vs. temperature
  - Some ductile materials experience brittle fracture – low temps.

Time- and rate-dependent behaviour



Anelasticity

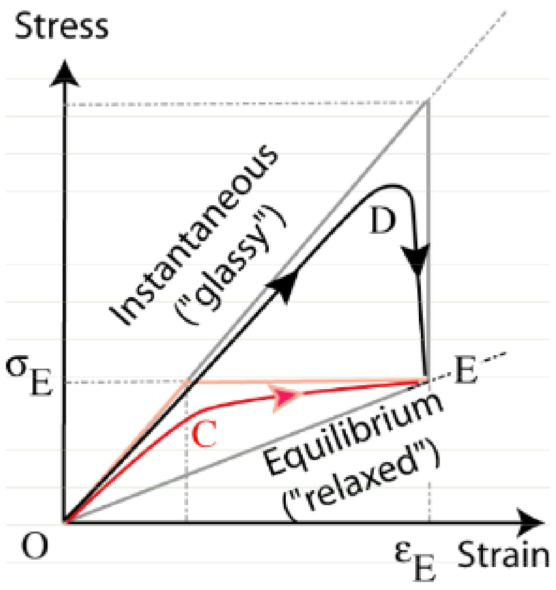
For nanocrystalline materials, elastic moduli measured using acoustic methods are higher than those measured in quasi-static uniaxial tension tests -> anelastic or plastic deformation is taking place.

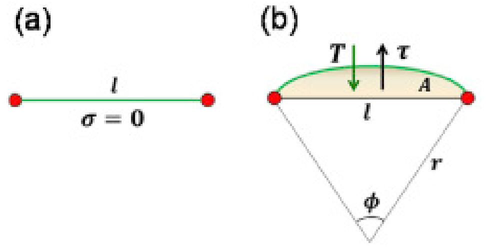
Atomistic simulations suggest that grain-boundary sliding and grain rotation play a role in nanocrystal deformation even at very low loads.

Anelasticity

Anelastic relaxation processes:

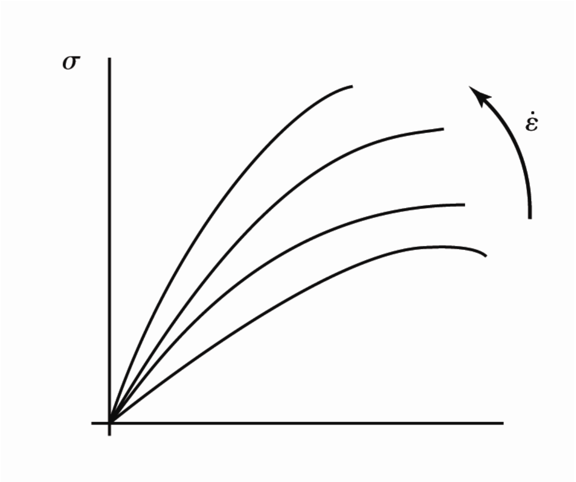
- Lead to hysteretic behavior in crystals
- Associated with dislocations and grain boundaries
- For thin metal films at room temperature, the modulus reduction can be up to 10%
- Dislocation segment pinned at two points can bow under an applied shear stress. If the stress is removed, the bowed segment will straighten under its own line tension.

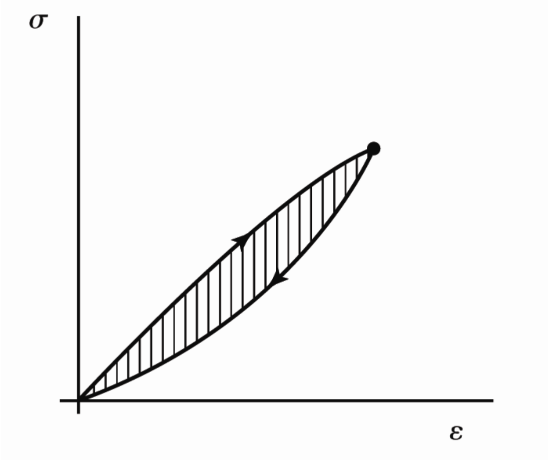




Viscoelasticity

- Time-dependent change of stress-strain behaviour is referred to as *viscoelasticity*
- Viscous: stress is proportional to the time rate of strain
- Elastic: stress is proportional to the strain
- In polymers, this behaviour is caused by the relative movement of entangled polymer chains leading to characteristic relaxation times





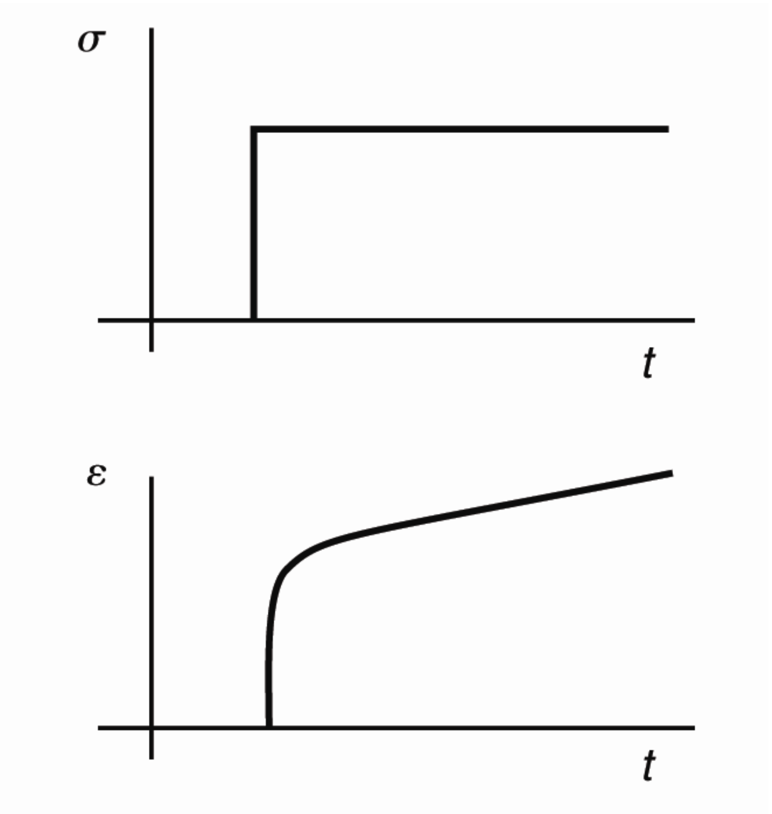
Viscoelastic creep

Action:

- Stress step followed by constant stress

Reaction:

- Instantaneous elastic followed by time-dependent deformation



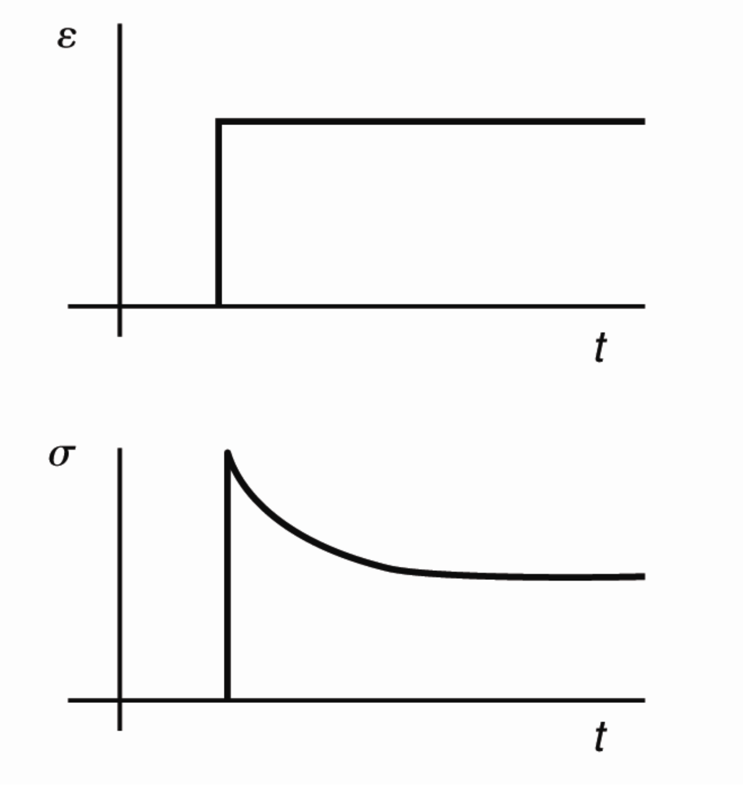
Stress relaxation

Action:

- Strain step followed by constant strain

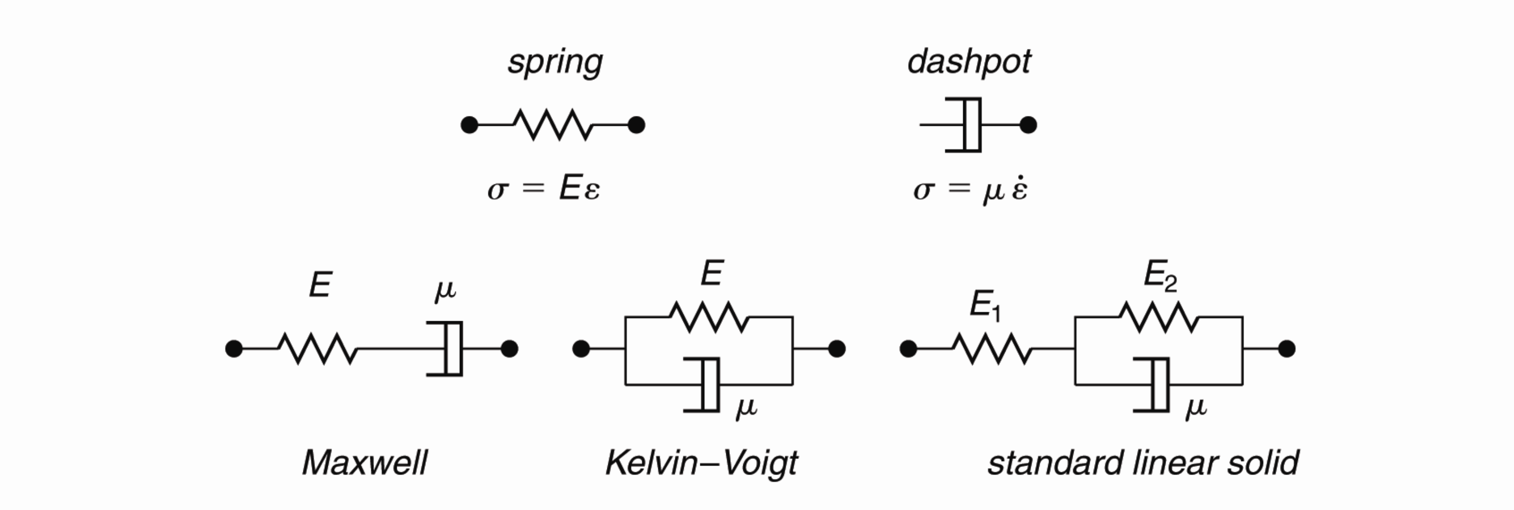
Reaction:

- Elastic response followed by time-dependent stress decrease

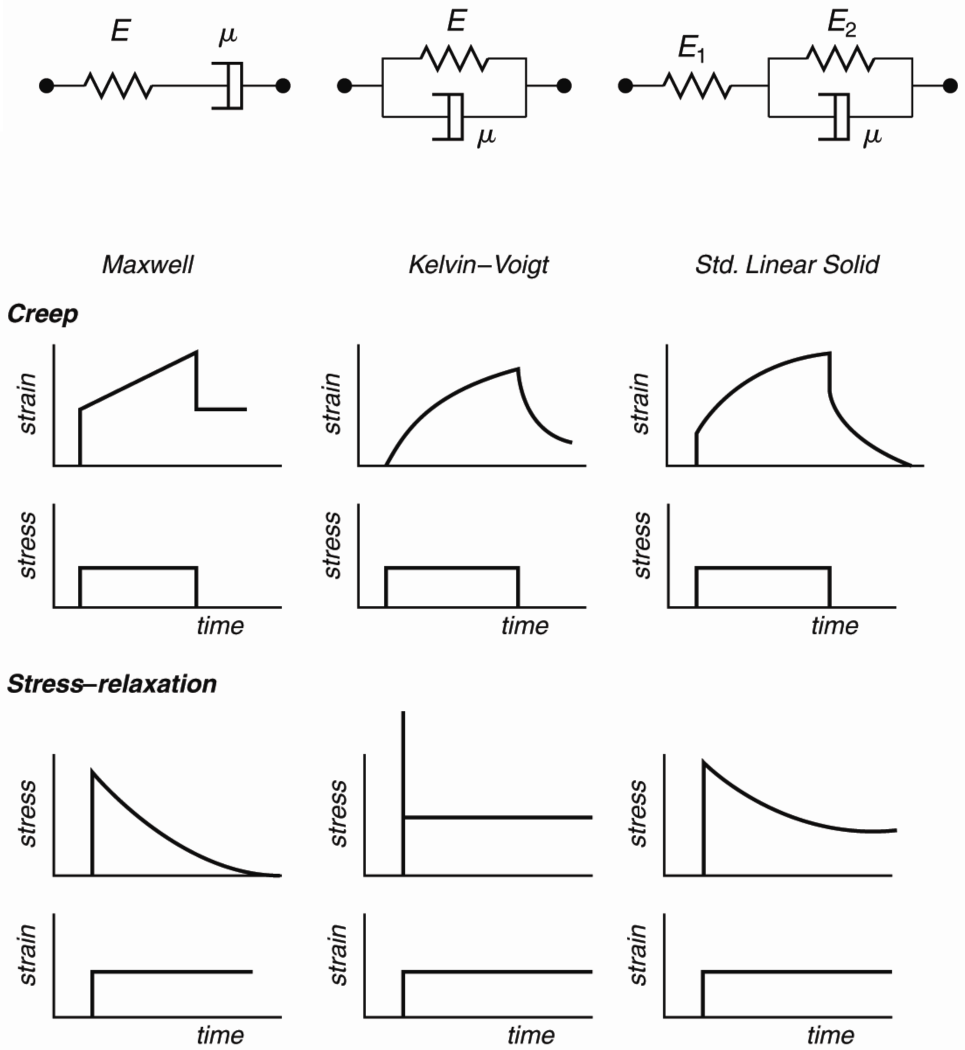


Linear viscoelastic models

- Phenomenological approach:
  - Combination of elastic springs and viscous dashpots
- The challenge is to find the simplest model that can adequately describe the experimentally observed behaviour



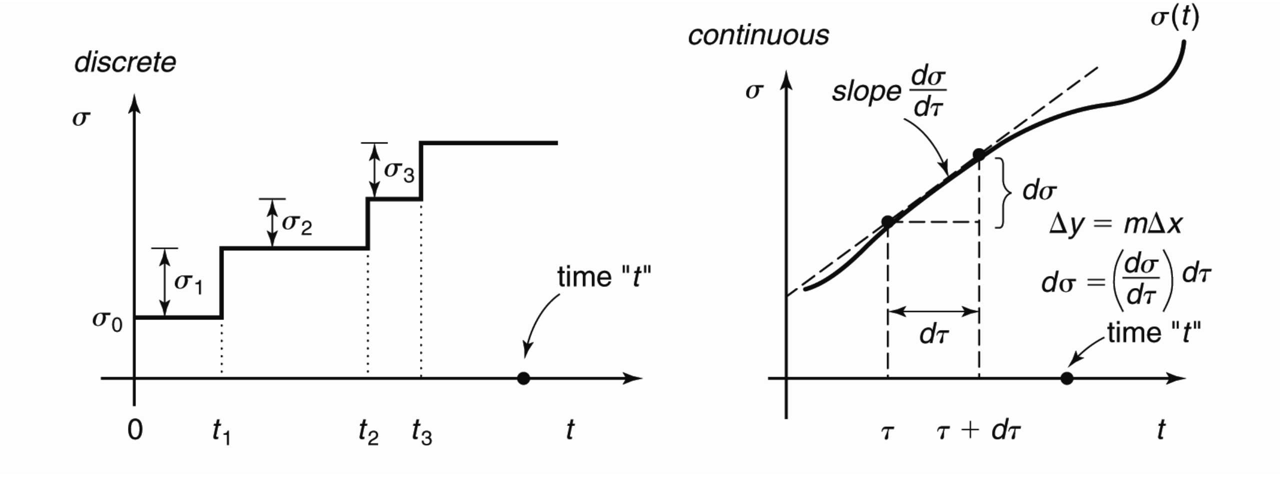
Linear viscoelastic models



Time integration

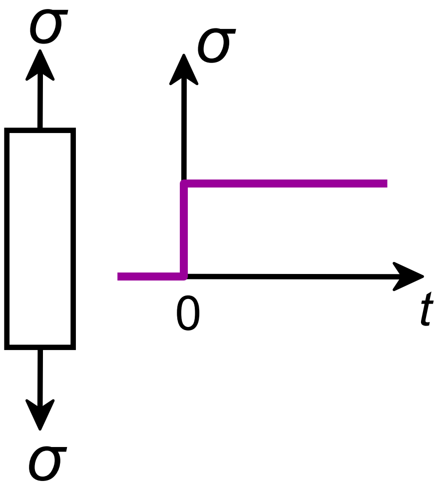
- Response depends on the entire time history of loading
- Principle of linear superposition
- Knowledge of creep and stress relaxation function allows detemining response to general load cases:

$$\varepsilon(t) = \int_{-\infty}^{t} \frac{d\sigma(\tau)}{d\tau} c(t-\tau)\, d\tau$$

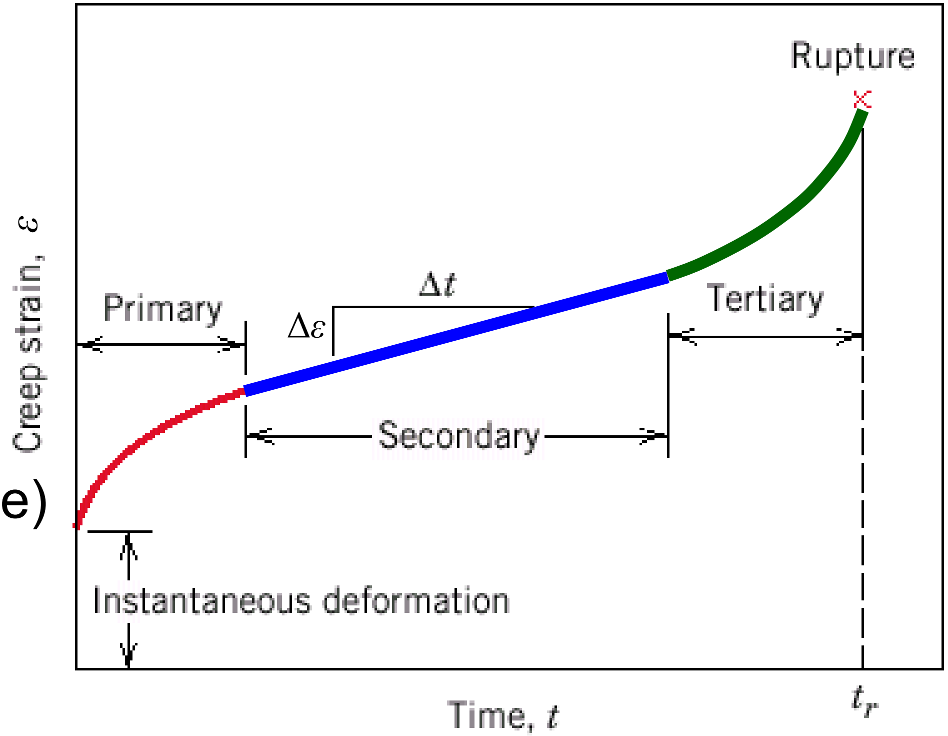


Creep

Measure deformation (strain) vs. time at constant stress



Occurs at elevated temperature for most metals, $T > 0.4\ T_m$ (in K)

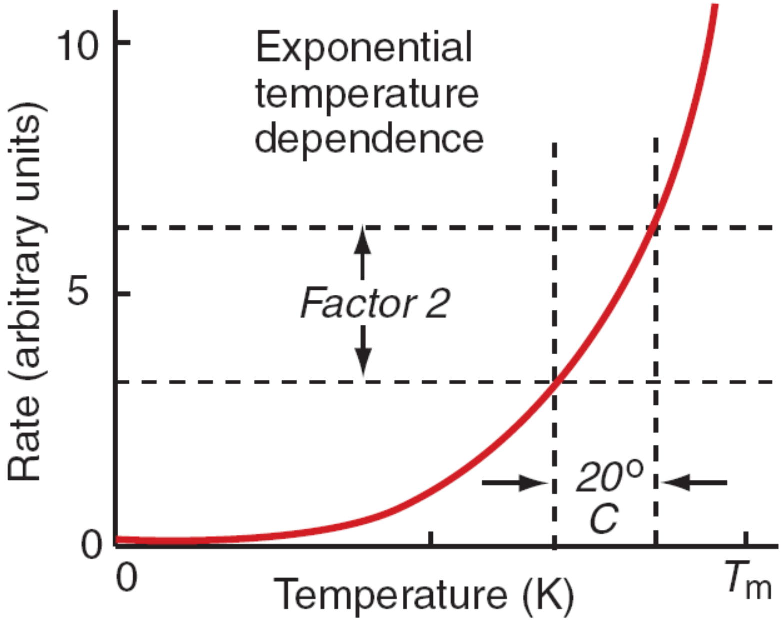


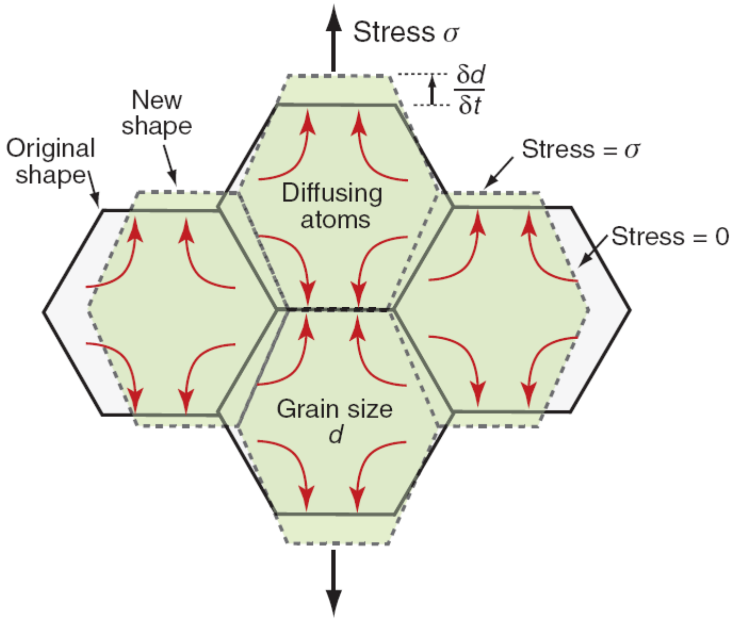
Adapted from Fig. 8.30, *Callister & Rethwisch 10e.*

Stages of Creep

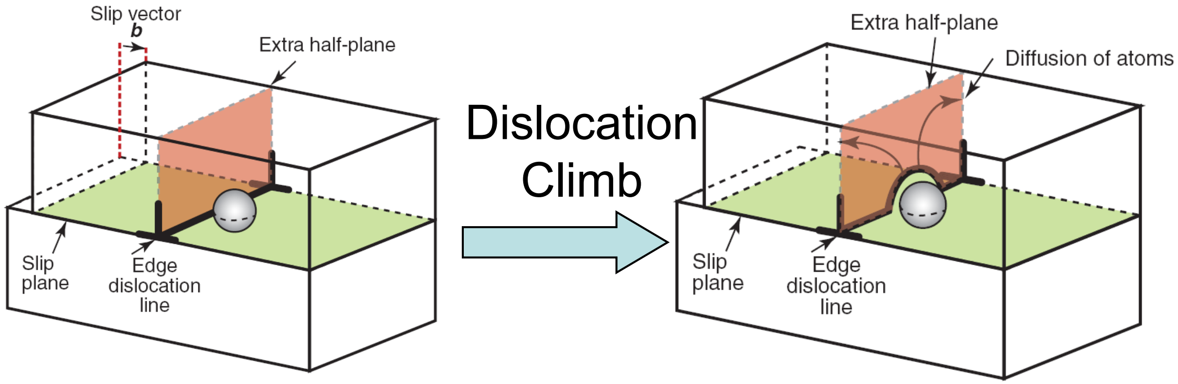
- Primary Creep: slope (creep rate) decreases with time.
- Secondary Creep: steady-state i.e., constant slope ($\Delta\varepsilon/\Delta t$).
- Tertiary Creep: slope (creep rate) increases with time, i.e. acceleration of rate.

Plastic creep mechanisms





Diffusional flow

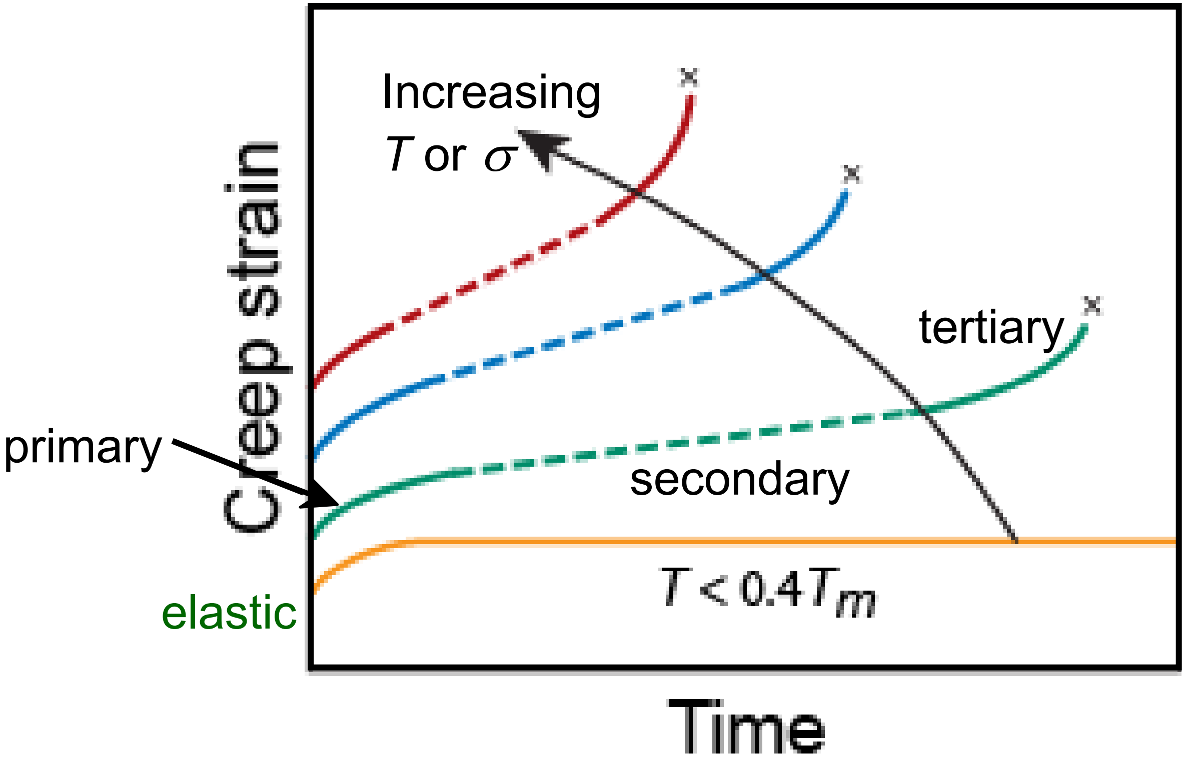


Dislocation Climb

Power-law creep

$$\dot{\varepsilon}_{ss} = A \times \sigma^{n} \times \exp-\left(\frac{Q}{RT}\right)$$

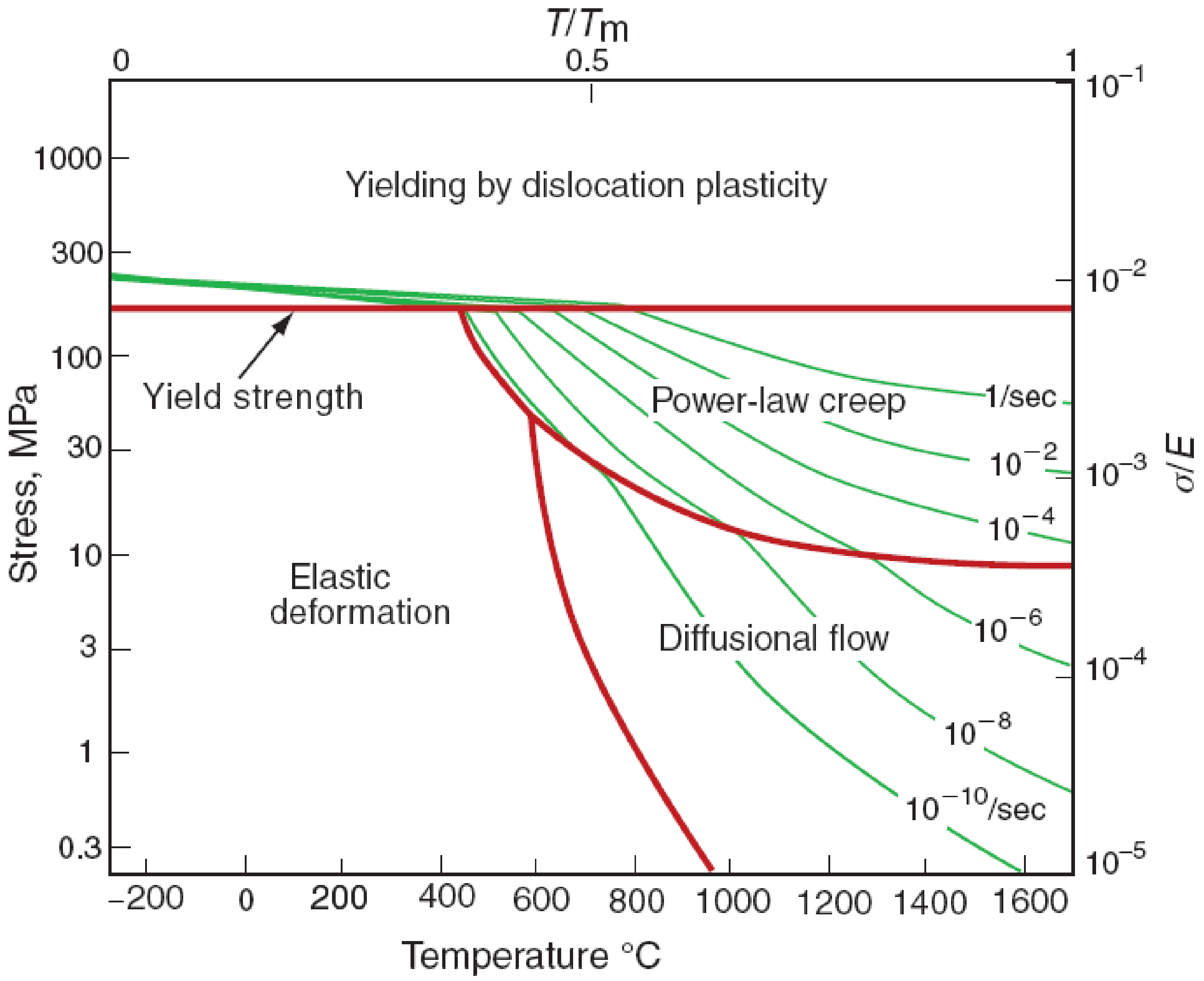
Creep: Temperature Dependence



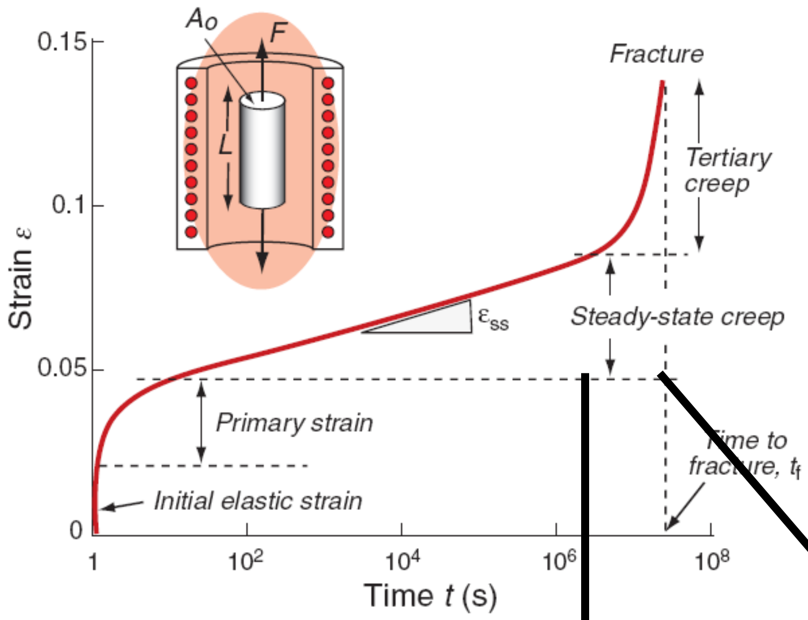
Figs. 8.31, *Callister & Rethwisch 10e.*

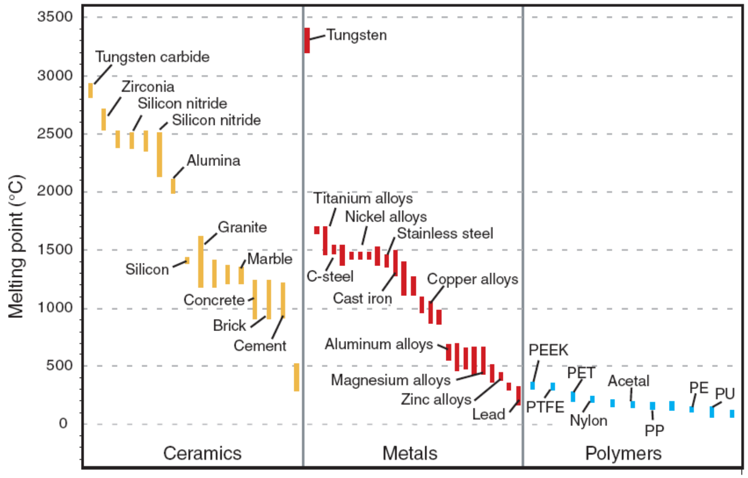
- Steady-state creep rate ($\dot{\varepsilon}_s$) increases with increasing T and σ
- Rupture lifetime (t_r) decreases with increasing T and σ

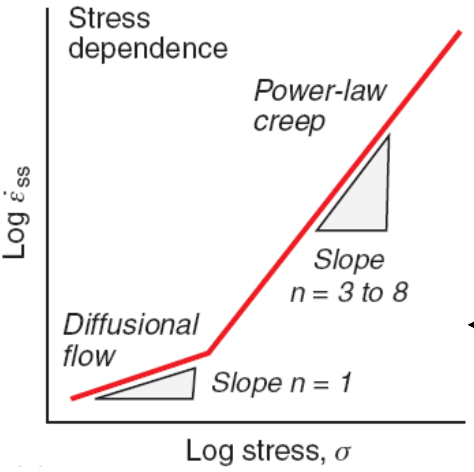
Deformation Mechanism Map



High temperature materials: Creep

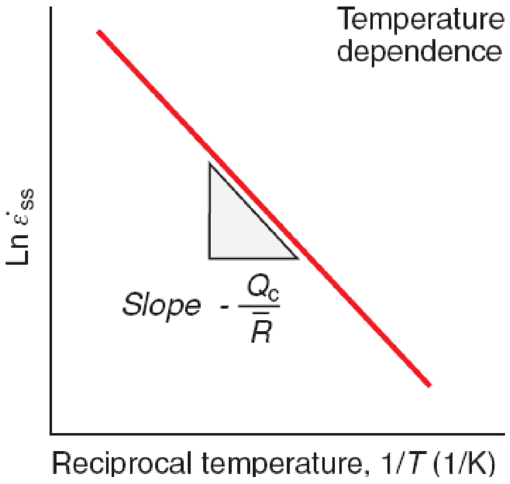






Mechanisms

Activation energy



Steady-State Creep Rate

- $\dot{\varepsilon}_s$ constant for constant T, σ
  - -- strain hardening is balanced by recovery
  - -- dependence of steady-state creep rate on T, σ

stress exponent (material parameter)

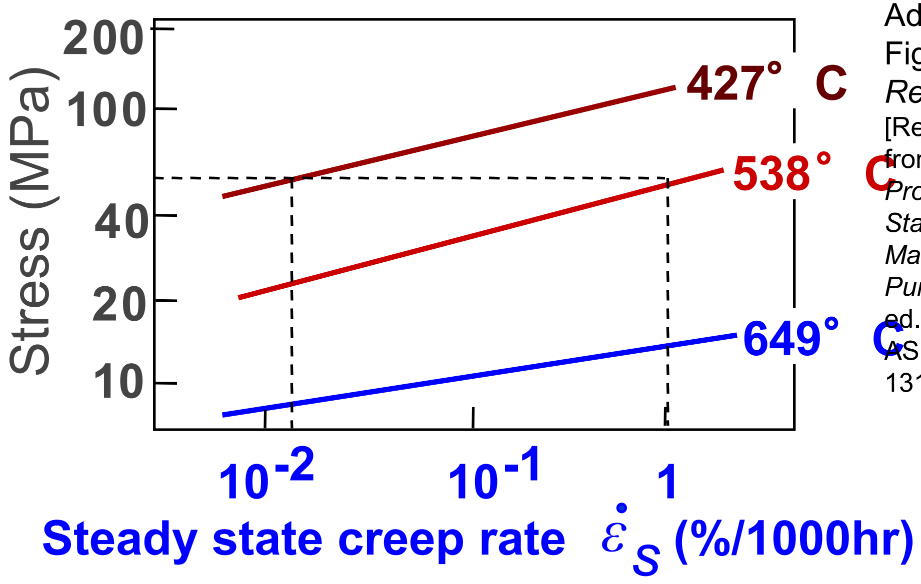
$$\dot{\varepsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

activation energy for creep (material parameter)

material const.

applied stress

- Steady-state creep rate increases with increasing T, σ



Adapted from Fig. 8.31, *Callister & Rethwisch 7e.* [Reprinted with permission from *Metals Handbook: Properties and Selection: Stainless Steels, Tool Materials, and Special Purpose Metals*, Vol. 3, 9th ed., D. Benjamin (Senior Ed.), ASM International, 1980, p. 131.]

Prediction of Creep Rupture Lifetime

Time to rupture, t_r

- Creep data for prolonged time (e.g., years) impractical to collect in lab
- Extrapolate from data collected for shorter time at higher T using Larson-Miller parameter, m, defined as

constant (normally ≈ 20)

$$T(C + \log t_r) = m$$

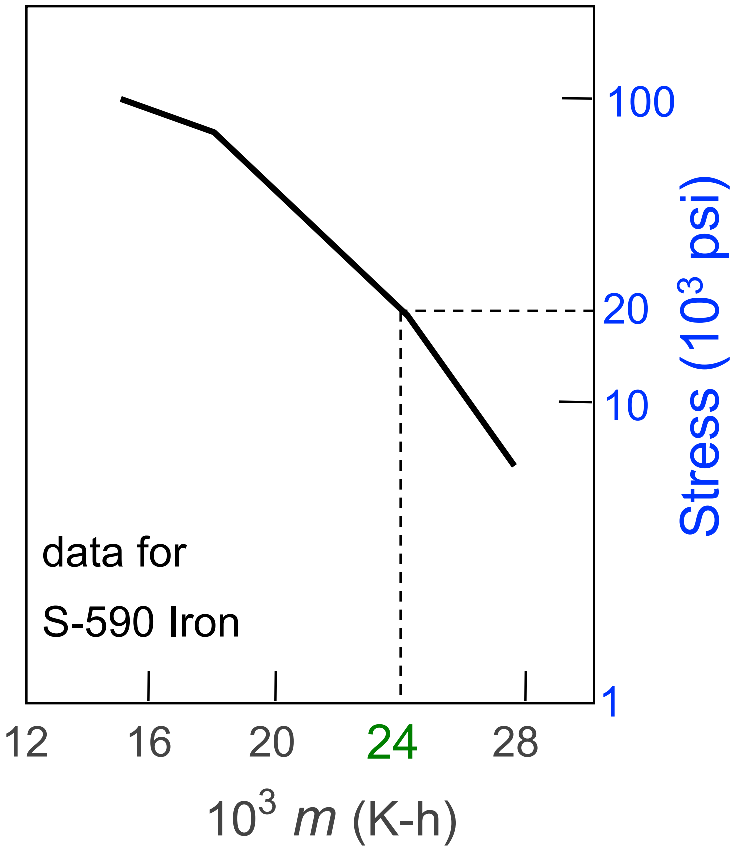
temperature

function of
applied stress

time to failure (rupture)

Prediction of Creep Rupture Lifetime (cont.)

- Plot of log stress vs. Larson-Miller parameter



Adapted from Fig. 8.34, *Callister & Rethwisch 10e.* (From F.R. Larson and J. Miller, *Trans. ASME*, **74**, 765 (1952). Reprinted by permission of ASME)

- Example: Estimate the rupture time t_r for S-590 Iron at T = 800° C and σ = 20,000 psi

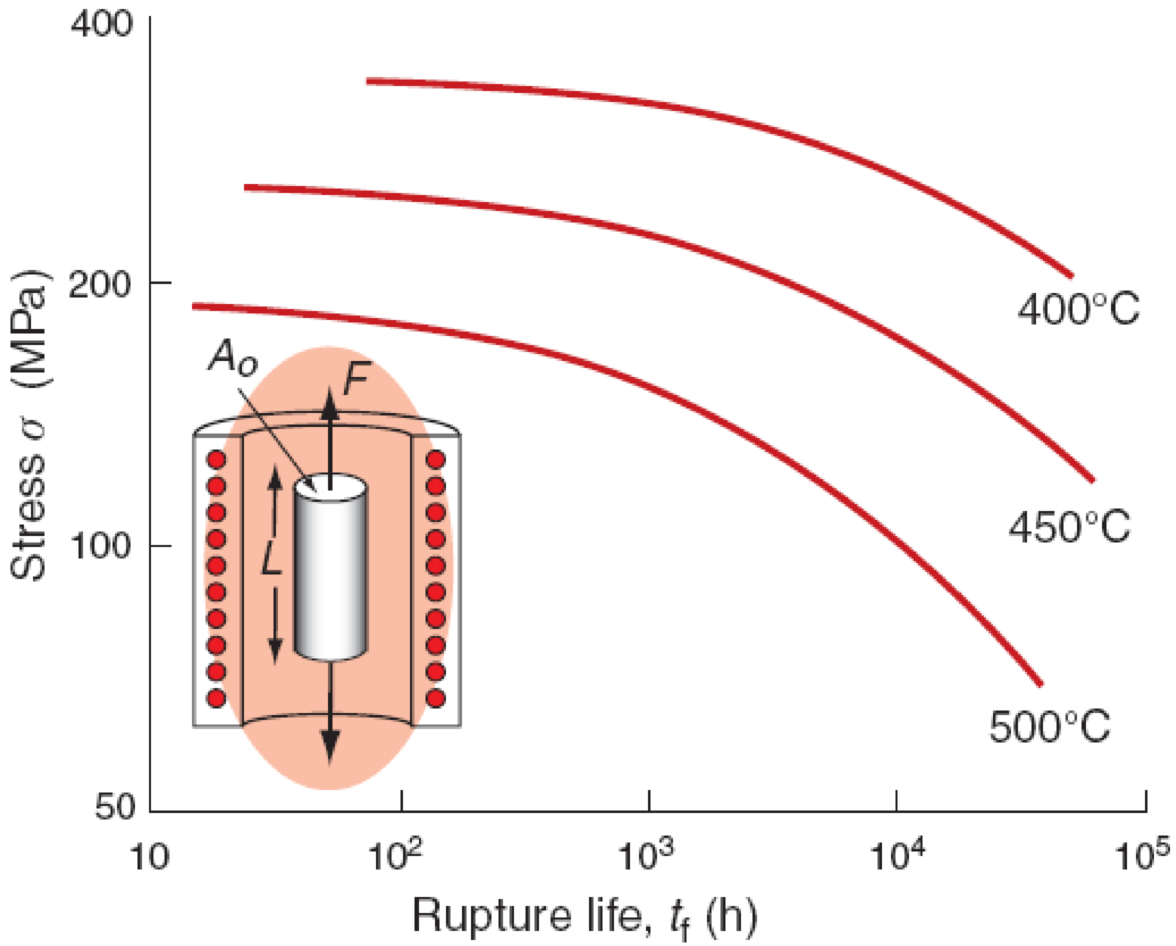
$$T = 800 + 273 = 1073 \text{ K}$$

$$T(C + \log t_r) = m$$

$$(1073 \text{ K})(20 + \log t_r) = 24\text{x}10^3$$

Ans: t_r = 233 hr

Creep rupture diagram

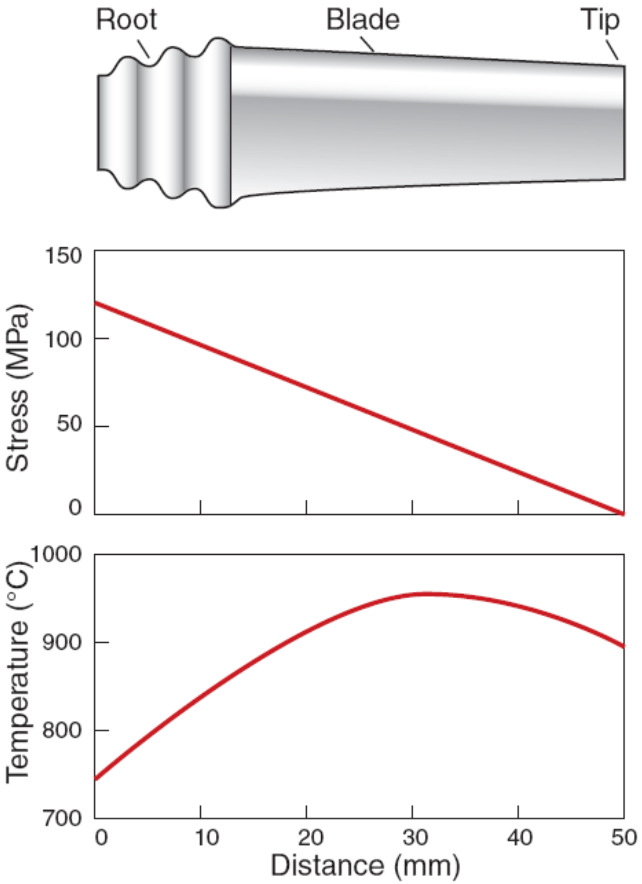


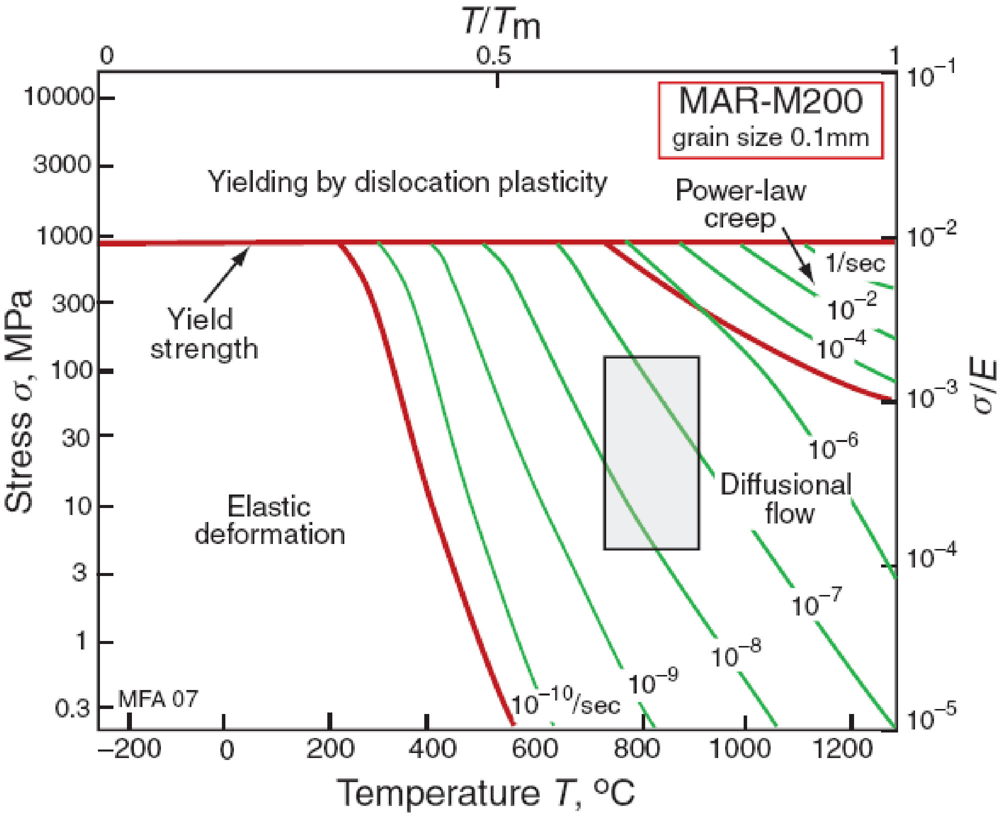
Monkman-Grant law:

$$\dot{\varepsilon} \times t_f = C$$

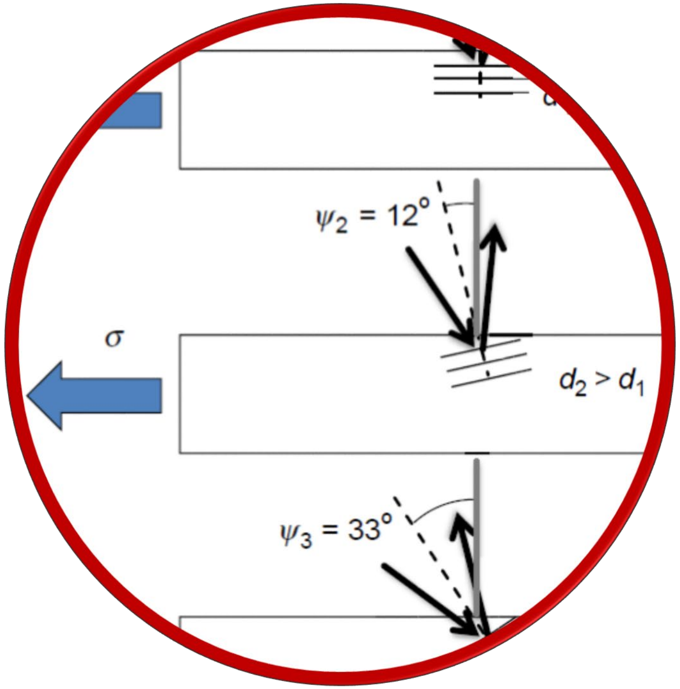
C ~ 0.05 - 3

Coping with creep!



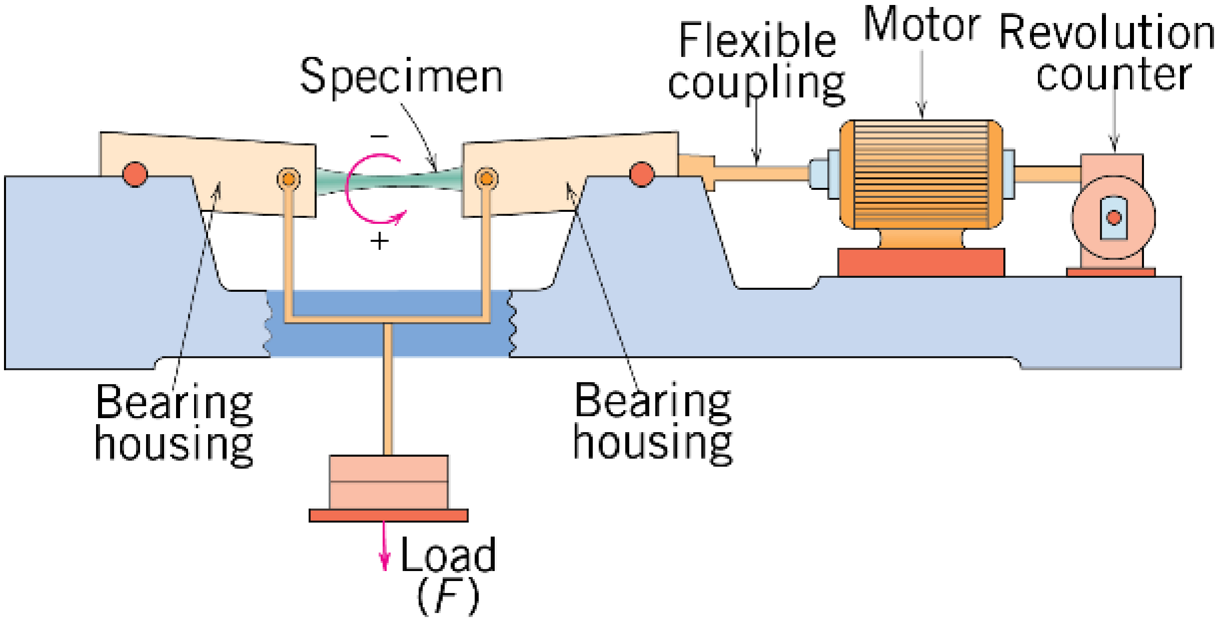


Cyclic loading



Fatigue Failure

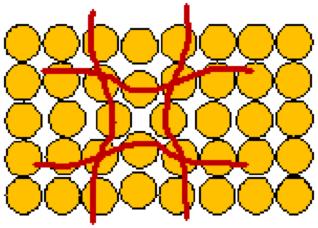
- Fatigue = failure under lengthy period of repeated stress or strain cycling
- Stress varies with time.
  -- key parameters are S, σ_m, and cycling frequency
- Key points: Fatigue...
  --can cause part failure, even though applied stress $\sigma_{max} < \sigma_y$.
  --responsible for ~ 90% of mechanical engineering failures.



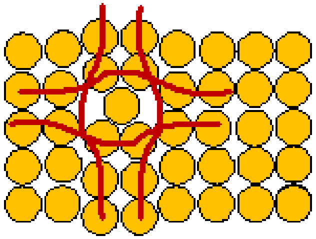
Schematic diagram of an apparatus for performing rotating-bending fatigue tests

Adapted from Fig. 8.19(a), *Callister & Rethwisch 10e.*

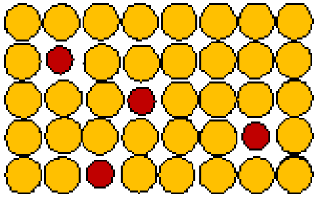
Microstructure (Defects)



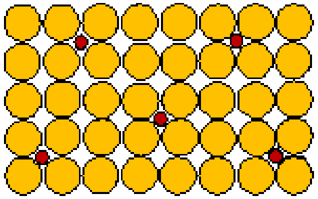
Vacancies:
vacant atomic sites



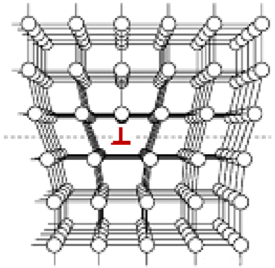
Self-interstitials:
extra atoms



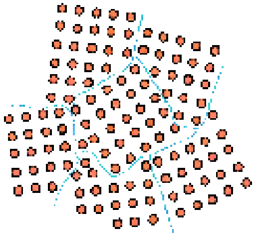
Impurities:
substitutional



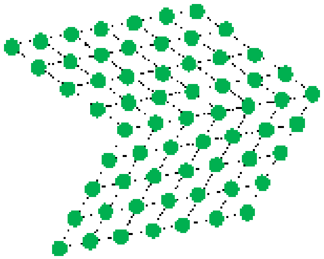
Impurities:
interstitial



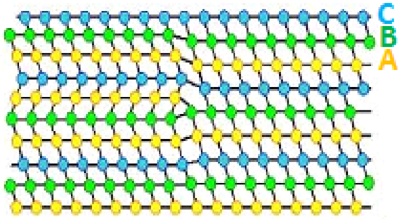
Dislocations



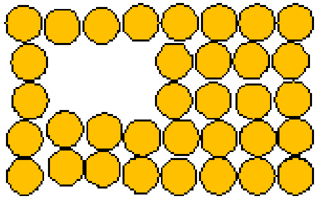
Grain boundaries



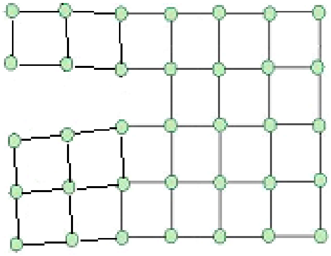
Twinning boundaries



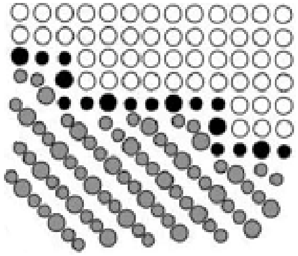
Stacking faults



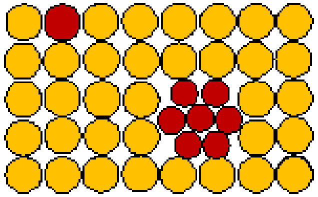
Microvoids



Microcracks

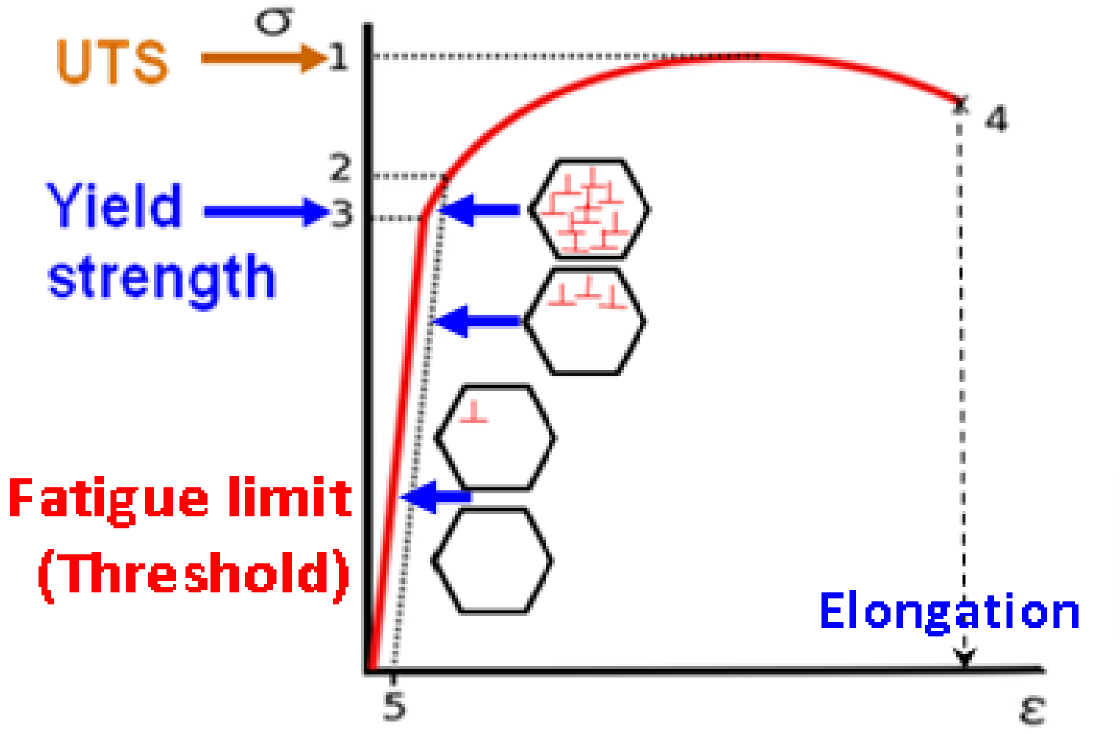


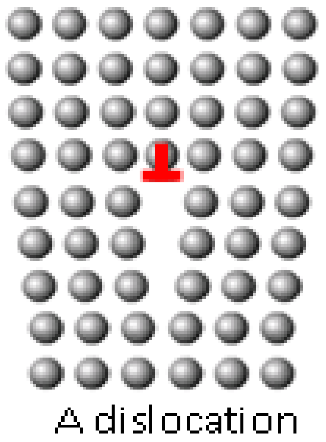
Phase interfaces



Second phase precipitates

Fatigue Mechanism



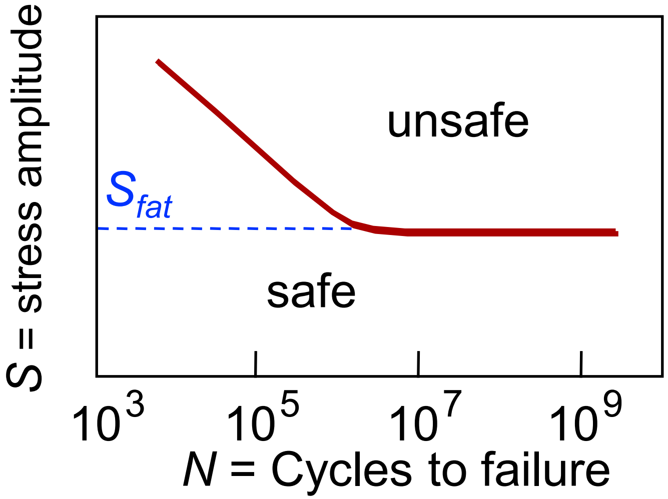


A dislocation

- A structurally perfect material (free of defects) would not suffer fatigue. Fatigue is always initiated at defect sites.

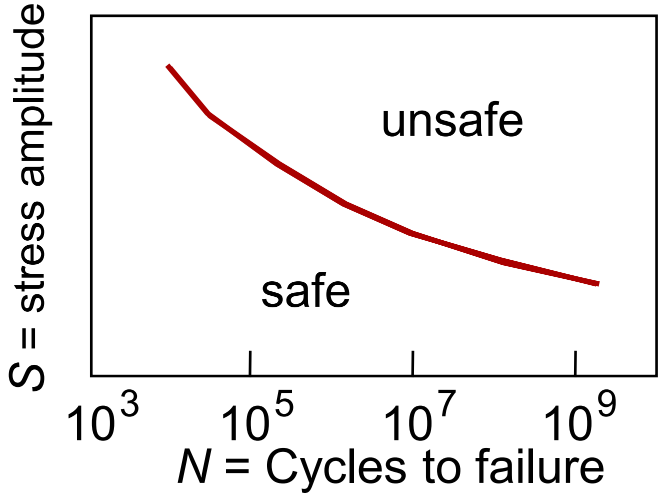
Types of Fatigue Behavior

- Fatigue data plotted as stress amplitude S vs. log of number N of cycles to failure.
- Two types of fatigue behavior observed
  - Fatigue limit, S_{fat}: no fatigue if $S < S_{fat}$
  - For some materials, there is no fatigue limit!
- Fatigue Life N_f = total number of stress cycles to cause fatigue failure at specified stress amplitude



case for steel (typ.)

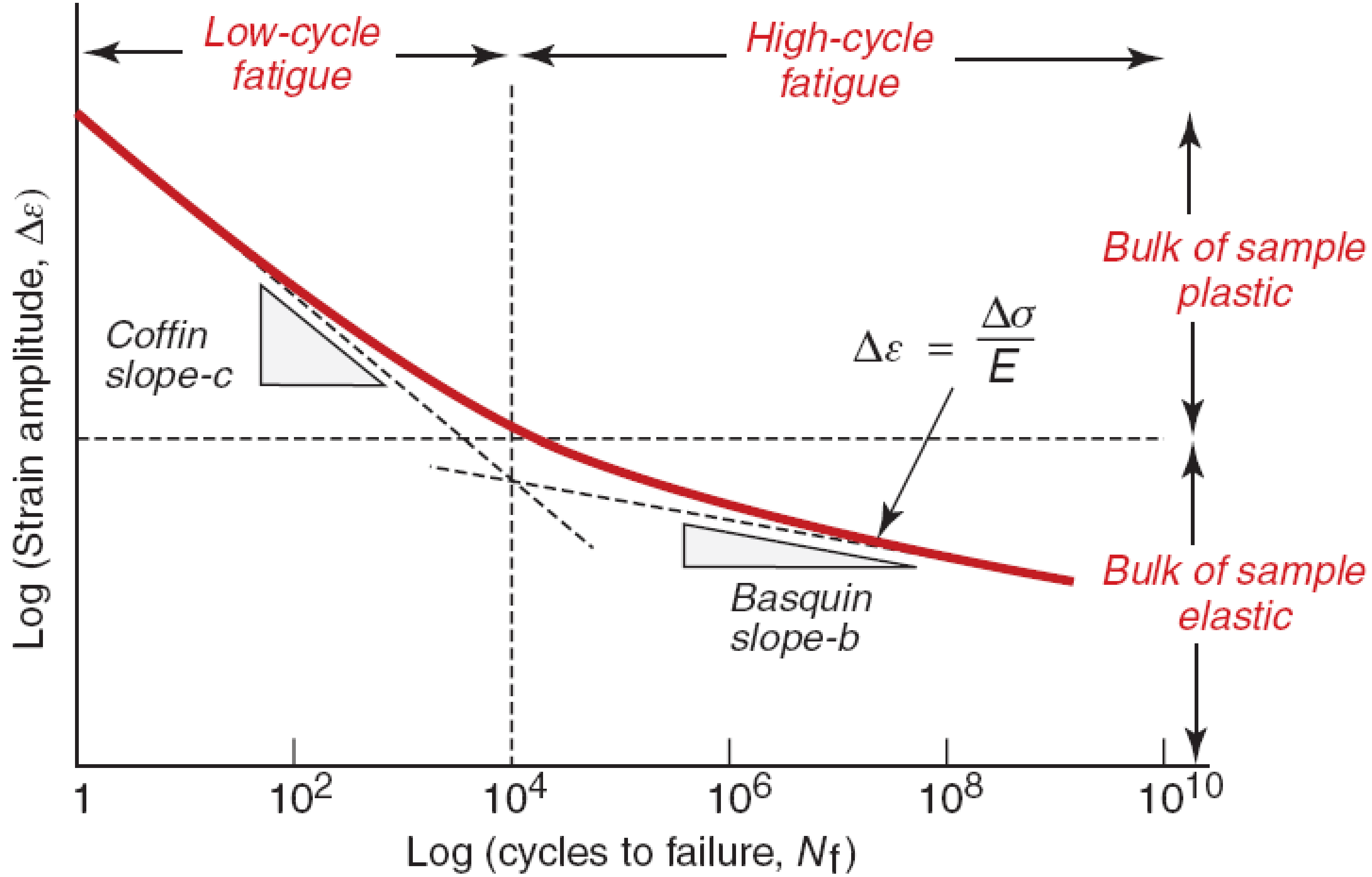
Adapted from Fig. 8.20(a), *Callister & Rethwisch 10e.*



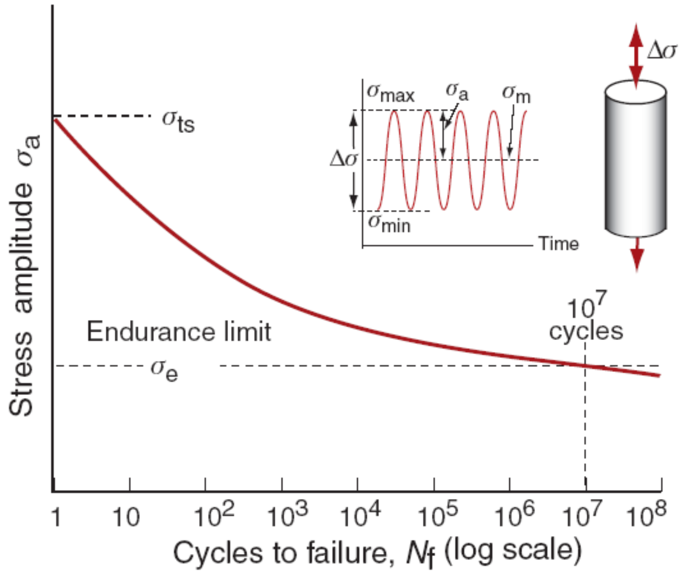
case for Al (typ.)

Adapted from Fig. 8.20(b), *Callister & Rethwisch 10e.*

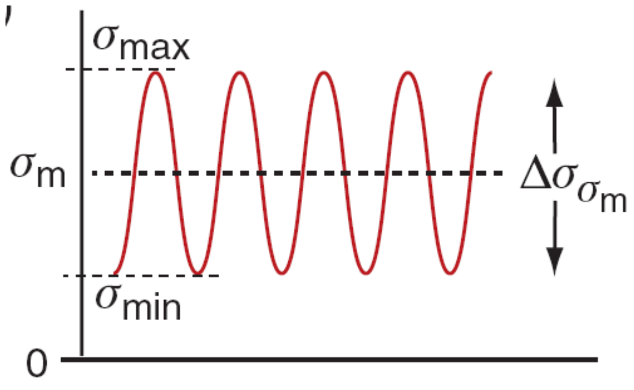
Material properties: fatigue



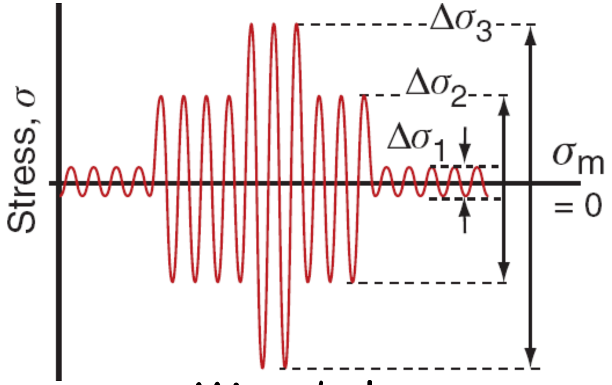
High cycle fatigue



Wöhler curve, S-N curve:
endurance limit

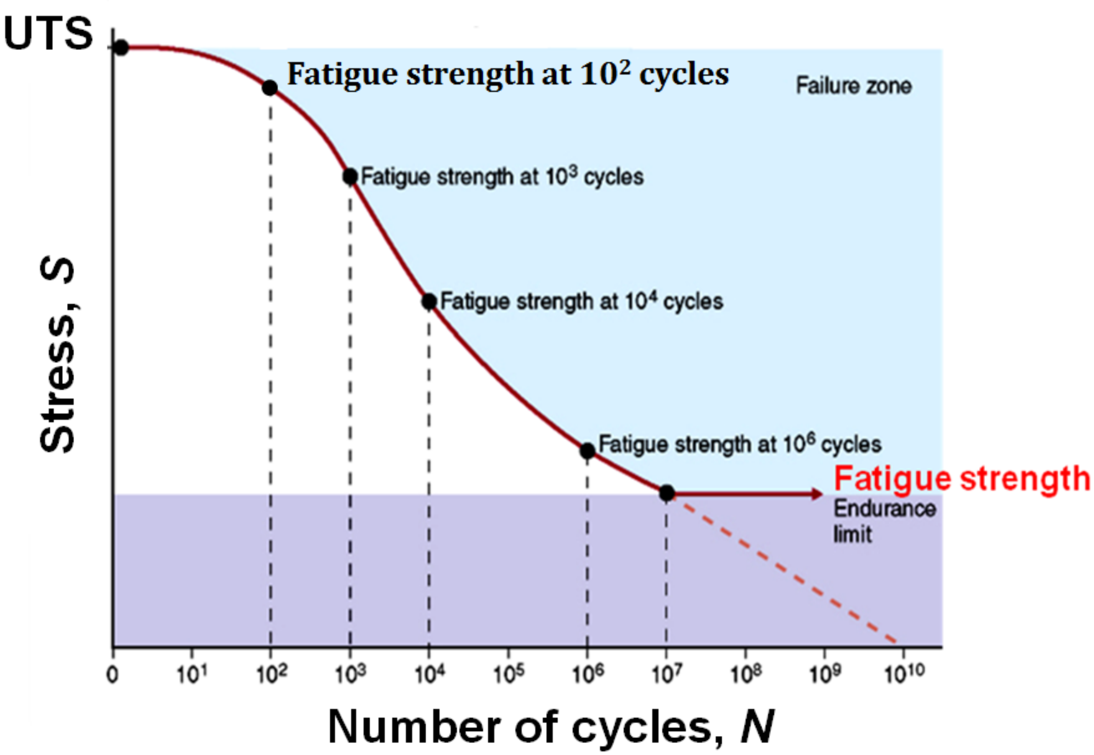


Goodman's law:
accounts for finite mean stress



Miner's law:
Accounts for variable amplitude

S-N Fatigue Curve



- ***S-N curve*** is a graph of the magnitude of a cyclic stress (*S*) against the logarithmic scale of cycles to failure (*N*).

Fatigue Limit & Fatigue Strength

- **Fatigue limit** or **endurance limit** refers to the maximal amplitude (or range) of cyclic stress that can be applied to a material without causing fatigue failure, regardless of how many cycles it is loaded.
- Most nonferrous alloys do not have a fatigue limit. For these materials, the number of 10^7 cycles has been widely used as the "infinitely" large number of cycles, and the stress level at which failure will occur at 10^7 cycles is called **Fatigue strength**.
- The fatigue strength is typically half of **UTS**, and even below the yield stress limit of the material:

$$\sigma_{\text{Fatigue}} \approx 0.5\ \sigma_{\text{UTS}}$$

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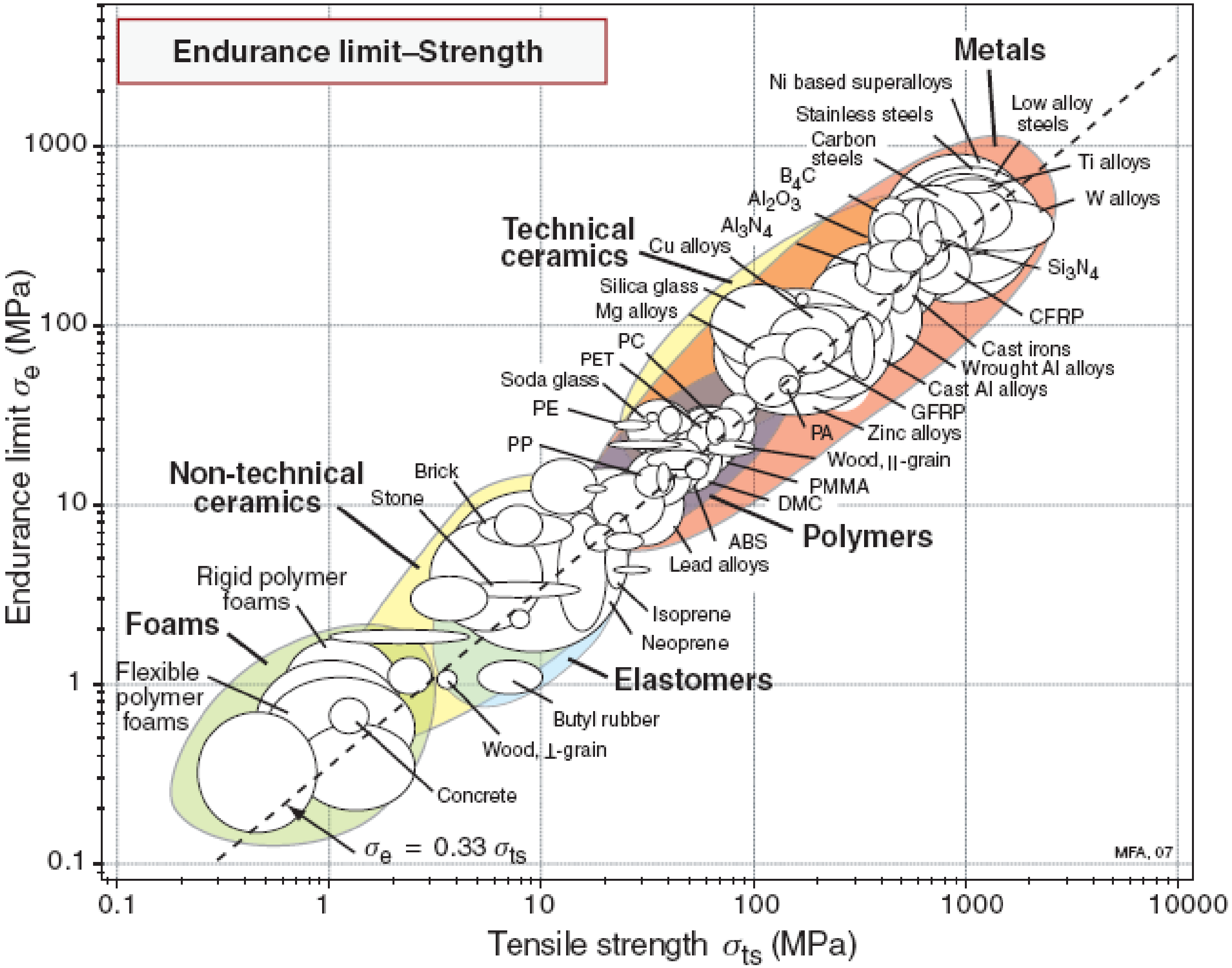
Fatigue Resistance

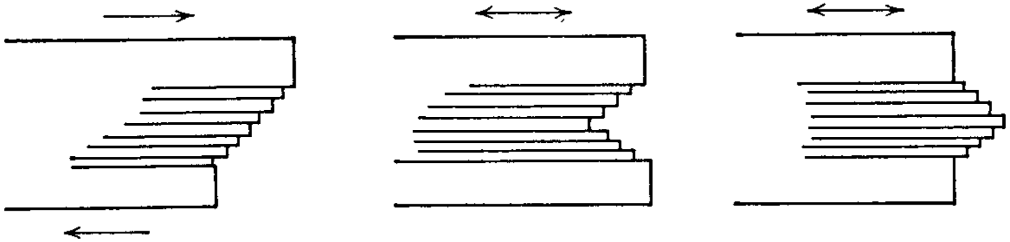
- Fatigue is the result of **accumulated micro-damage** (deformation). Hence, a resistance to deformation means a good resistance to fatigue.
- The optimal fatigue crack resistance is the combination of **high elastic modulus. high yield strength**, **and high fracture toughness.**

$$\sigma_{\text{Fatigue}} \approx 0.5\ \sigma_{\text{UTS}} \propto E \propto \sigma_{\text{y}}$$

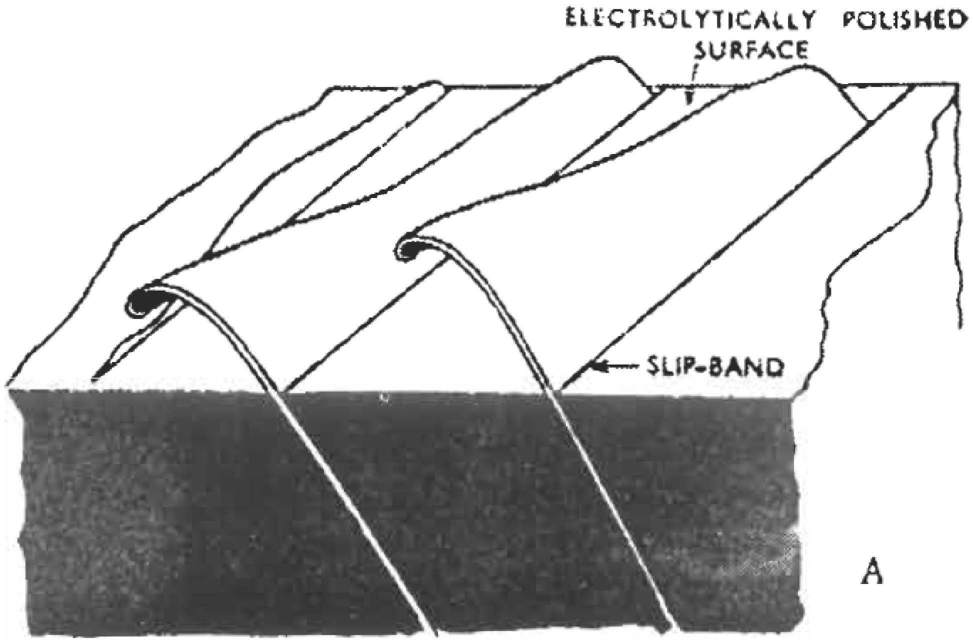
Endurance limit



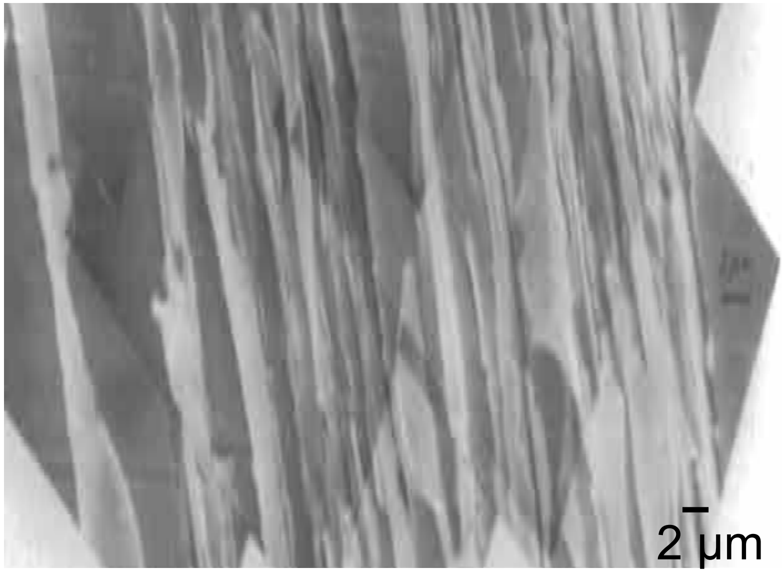
Origins of fatigue: Crack initiation



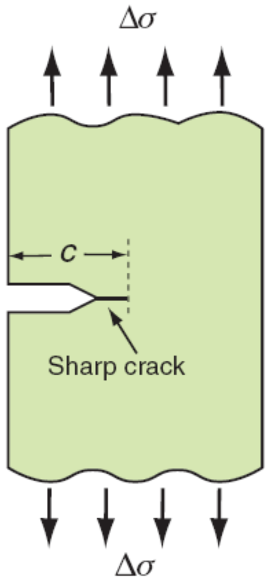
Wood's model (1959):
Intrusions & extrusions

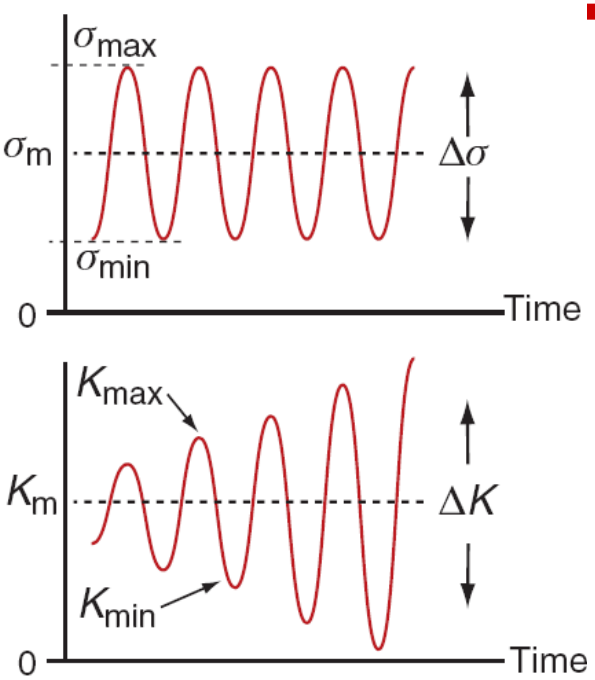


Macro-PSBs in a
Cu single crystal [Ma and Laird (1989)]

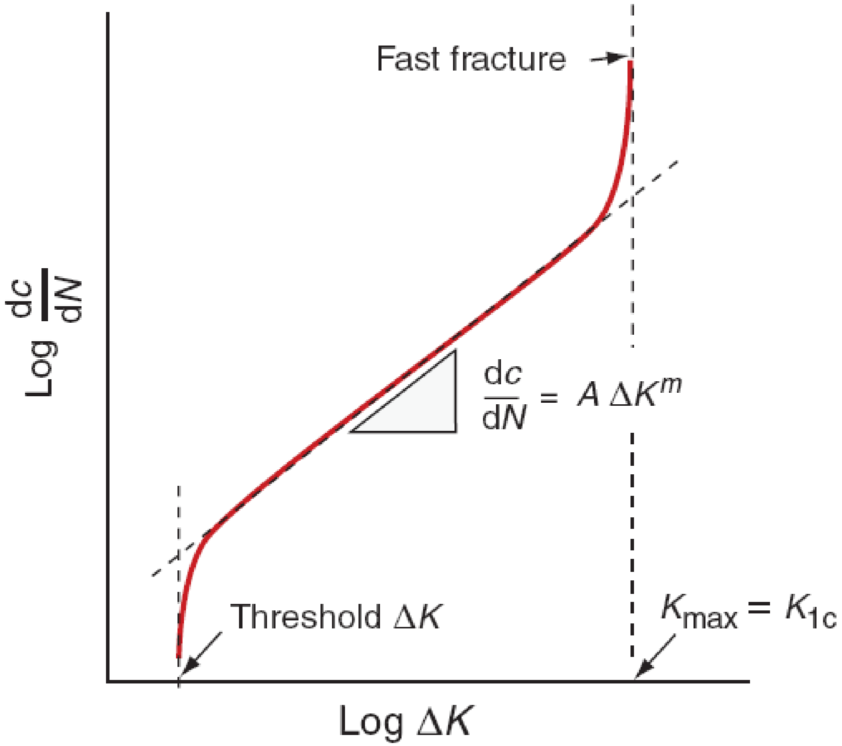


Fatigue loading of cracked components: Paris law





Characterization of
fatigue crack propagation:
Fracture mechanics approach



Paris' law: m~2-4 (metals)

$$\frac{dc}{dN} = A(\Delta K)^m$$

Fatigue life prediction: Living with cracks!

$$\frac{dc}{dN} = A(\Delta K)^m \qquad \Delta K = K_{\max} - K_{\min} = \Delta\sigma\sqrt{\pi c}$$

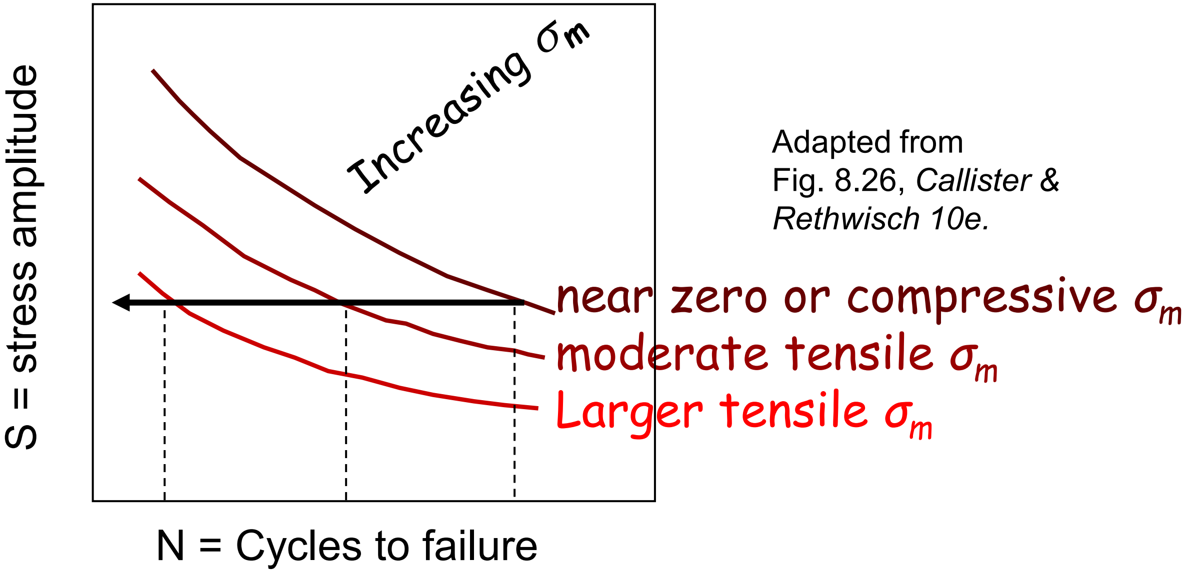
$$\frac{dc}{dN} = A(\Delta\sigma)^m (\pi c)^{m/2}$$

$$N_t = \frac{1}{A*(\Delta\sigma)^m} \int_{c_i}^{c^*} \left(\frac{dc}{(\pi c)^{m/2}} \right)$$

c* - critical crack length; value at which fast fracture will occur
c_i – initial crack length

Improving Fatigue Life

- Three general techniques to improve fatigue life
  1. Reducing magnitude of mean stress
  2. Surface treatments
  3. Design changes



Adapted from
Fig. 8.26, *Callister & Rethwisch 10e.*

Decreasing mean stress increases fatigue life

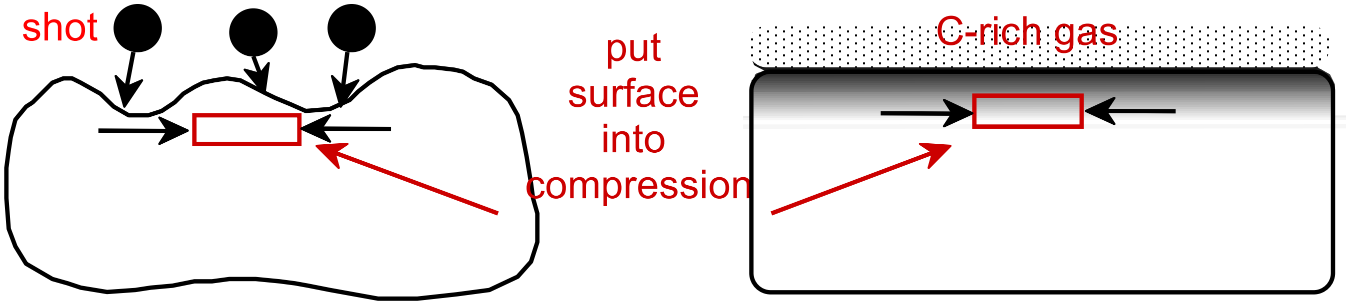
Improving Fatigue Life

- Three general techniques to improve fatigue life
  1. Reducing magnitude of mean stress
  2. Surface treatments
  3. Design changes

Imposing compressive surface stresses increases surface hardness – suppresses surface cracks from growing

Method 1: shot peening

Method 2: carburizing



surface compressive stress due to plastic deformation of outer surface layer

surface compressive stress due to carbon atoms diffusing into outer surface layer

Improving Fatigue Life

- Three general techniques to improve fatigue life
  1. Reducing magnitude of mean stress
  2. Surface treatments
  3. Design changes

Remove stress concentrators

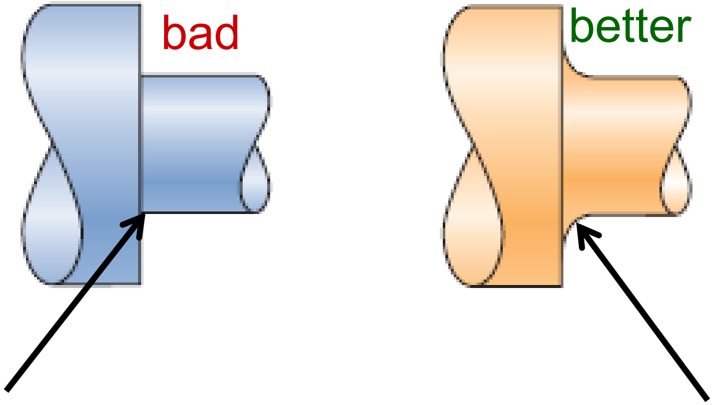
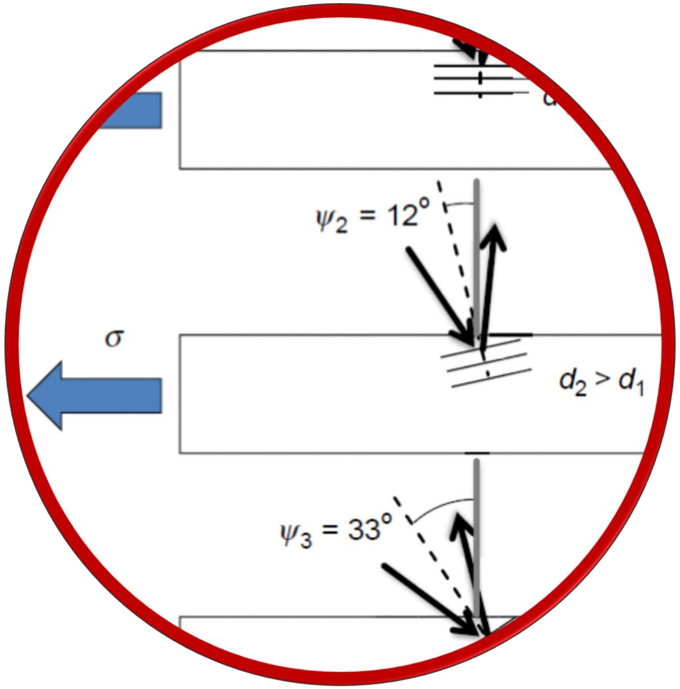


Fig. 8.27, *Callister & Rethwisch 10e.*

sharp corner – point of stress concentration

rounding corner reduces stress concentration

Stress in thin films



Composites

ISSUES TO ADDRESS...

- What are the classes and types of composites?
- What are the advantages of using composite materials?
- How do we predict the stiffness and strength of the various types of composites?

Composite

- Combination of two or more individual materials
- Design goal: obtain a more desirable combination of properties (principle of combined action)
  - e.g., low density and high strength

Terminology/Classification

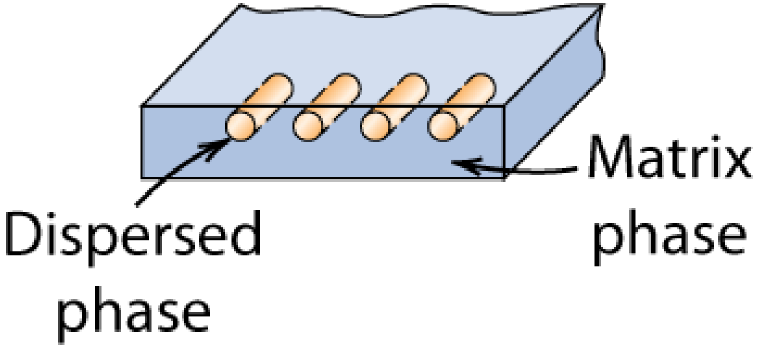
- Composite:

  -- Multiphase material that is artificially made.

- Phase types:

  -- Matrix - is continuous

  -- Dispersed - is discontinuous and surrounded by matrix



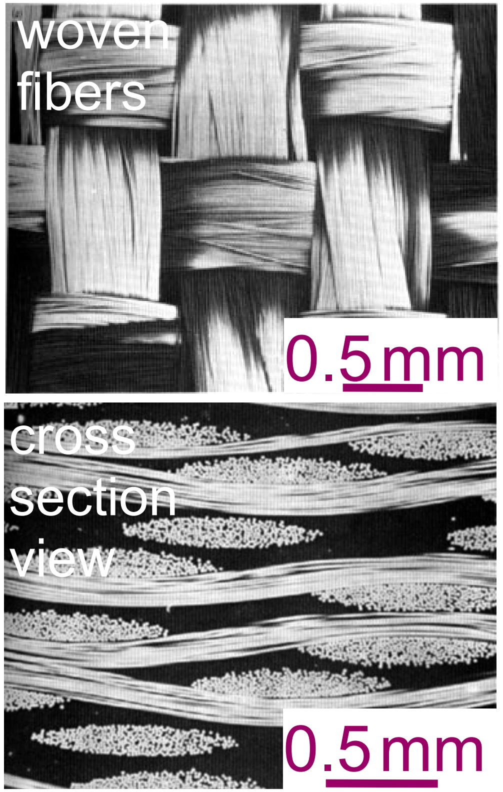
Adapted from Fig. 16.1(a),
Callister & Rethwisch 10e.

Terminology/Classification

- Matrix phase:
  - -- Purposes are to:
    - - transfer stress to dispersed phase
    - - protect dispersed phase from environment
  - -- Types: MMC, CMC, PMC

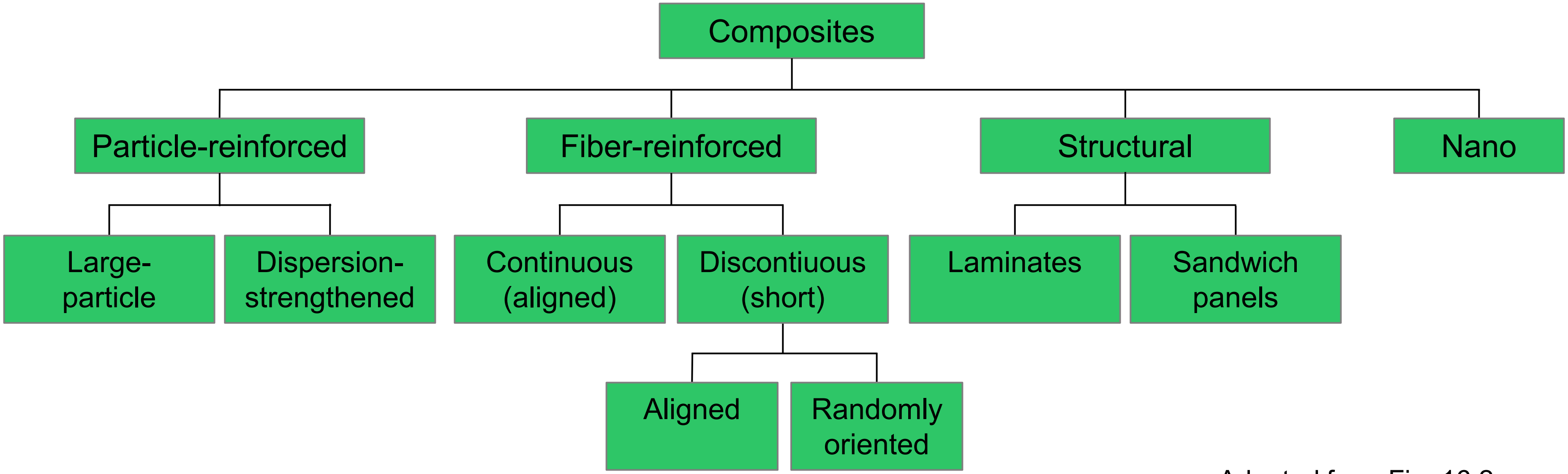
metal ceramic polymer

- Dispersed phase:
  - -- Purpose:
    - MMC: increase σ_y, TS, creep resist.
    - CMC: increase K_{Ic}
    - PMC: increase E, σ_y, TS, creep resist.
  - -- Types: particle, fiber, structural



Reprinted with permission from D. Hull and T.W. Clyne, *An Introduction to Composite Materials*, 2nd ed., Cambridge University Press, New York, 1996, Fig. 3.6, p. 47.

Classification of Composites



- Composites
  - Particle-reinforced
    - Large-particle
    - Dispersion-strengthened
  - Fiber-reinforced
    - Continuous (aligned)
    - Discontiuous (short)
      - Aligned
      - Randomly oriented
  - Structural
    - Laminates
    - Sandwich panels
  - Nano

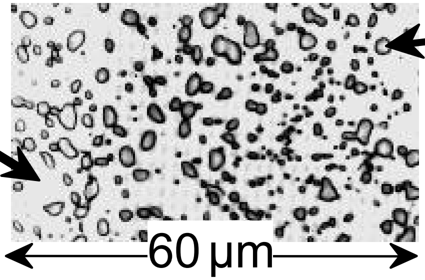
Adapted from Fig. 16.2, *Callister & Rethwisch 10e*.

Classification: Particle-Reinforced (i)

Particle-reinforced | Fiber-reinforced | Structural

- Examples:
  - Spheroidite steel

matrix:
ferrite (α)
(ductile)

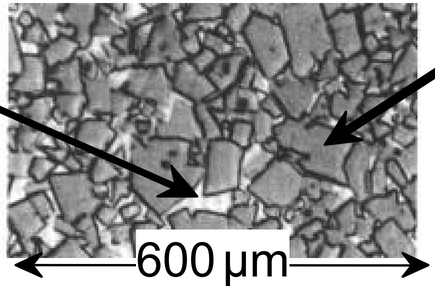


particles:
cementite
(Fe_3C)
(brittle)

Fig. 10.19, *Callister & Rethwisch 10e*.
(Copyright 1971 by United States Steel Corporation.)

- WC/Co cemented carbide

matrix:
cobalt
(ductile,
tough)

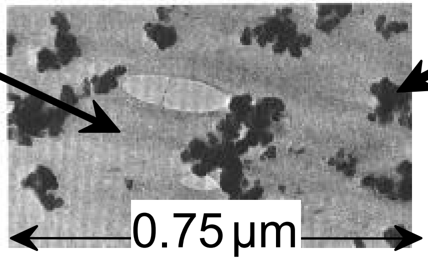


particles:
WC
(brittle,
hard)

Fig. 16.4, *Callister & Rethwisch 10e*.
(Courtesy of Carboloy Systems Department, General Electric Company.)

- Automobile tire rubber

matrix:
rubber
(compliant)



particles:
carbon
black
(stiff)

Fig. 16.5, *Callister & Rethwisch 10e*.
(Courtesy of Goodyear Tire and Rubber Company.)

Classification: Particle-Reinforced (ii)

Particle-reinforced Fiber-reinforced Structural

Concrete – gravel + sand + cement + water

- Why sand *and* gravel? Sand fills voids between gravel particles

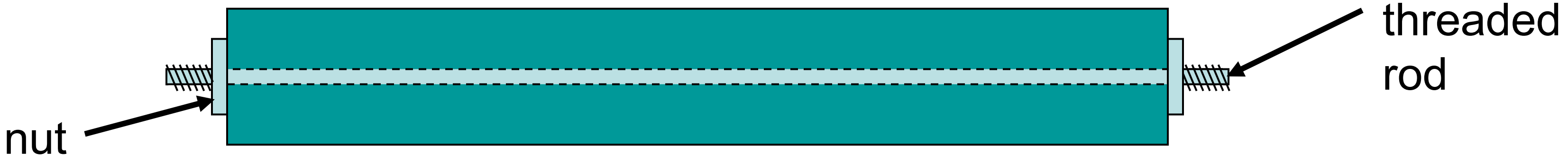
Reinforced concrete – Reinforce with steel rebar or remesh

- increases strength - even if cement matrix is cracked

Prestressed concrete

- Rebar/remesh placed under tension during setting of concrete
- Release of tension after setting places concrete in a state of compression
- To fracture concrete, applied tensile stress must exceed this compressive stress

Posttensioning – tighten nuts to place concrete under compression



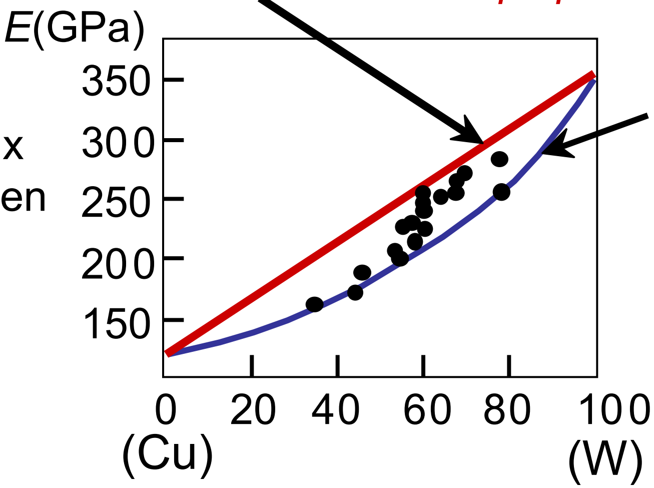
Classification: Particle-Reinforced (iii)

Particle-reinforced Fiber-reinforced Structural

- Elastic modulus, E_c, of composites:
  - -- two “rule of mixture” extremes:

upper limit: $E_c = V_m E_m + V_p E_p$

Data:
Cu matrix
w/tungsten
particles



lower limit:

$$\frac{1}{E_c} = \frac{V_m}{E_m} + \frac{V_p}{E_p}$$

Fig. 16.3, *Callister & Rethwisch 10e*. (Reprinted with permission from R. H. Krock, *ASTM Proceedings*, Vol. 63, 1963. Copyright ASTM International, 100 Barr Harbor Drive, West Conschohocken, PA 19428.)

- Application to other properties:
  - -- Electrical conductivity, σ_e: Replace E’s in equations with σ_e’s.
  - -- Thermal conductivity, k: Replace E’s in equations with k’s.

Classification: Fiber-Reinforced (i)

Particle-reinforced | **Fiber-reinforced** | Structural

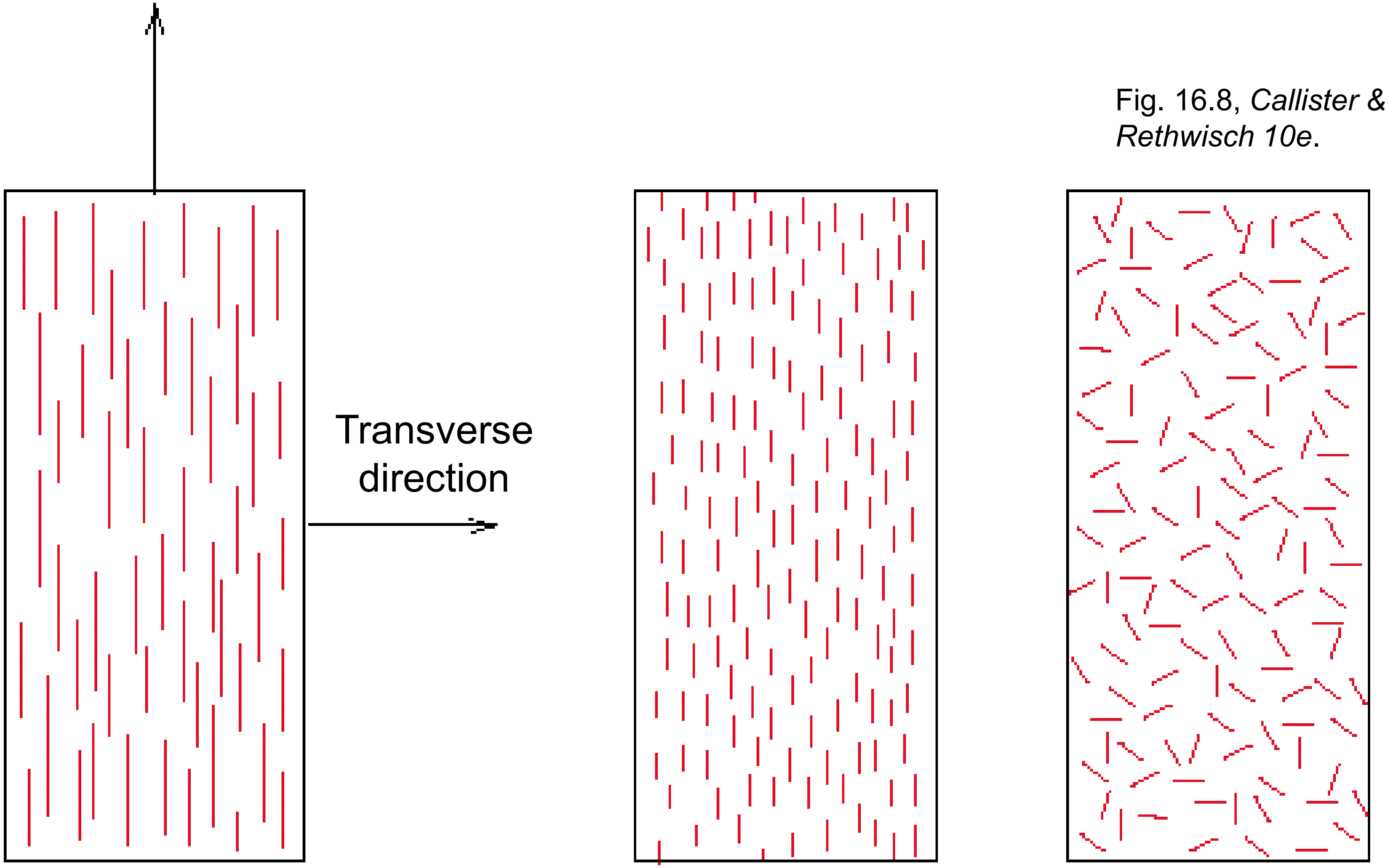
- Fibers very strong in tension
  - Provide significant strength improvement to the composite
  - Ex: fiber-glass - continuous glass filaments in a polymer matrix
    - Glass fibers
      - strength and stiffness
    - Polymer matrix
      - holds fibers in place
      - protects fiber surfaces
      - transfers load to fibers

Classification: Fiber-Reinforced (ii)

Particle-reinforced | **Fiber-reinforced** | Structural

- Fiber Types
  - Whiskers - thin single crystals - large length to diameter ratios
    - graphite, silicon nitride, silicon carbide
    - high crystal perfection – extremely strong, strongest known
    - very expensive and difficult to disperse
  - Fibers
    - polycrystalline or amorphous
    - generally polymers or ceramics
    - Ex: alumina, aramid, E-glass, boron, UHMWPE
  - Wires
    - metals – steel, molybdenum, tungsten

Fiber Alignment



Longitudinal direction

Transverse direction

Fig. 16.8, *Callister & Rethwisch 10e*.

aligned
continuous

aligned

random

discontinuous

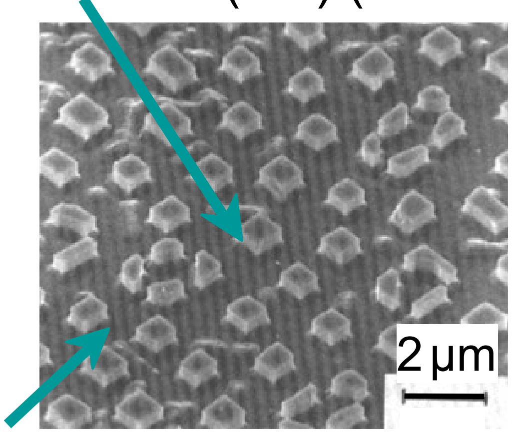
Classification: Fiber-Reinforced (iii)

Particle-reinforced | **Fiber-reinforced** | Structural

- Aligned Continuous fibers
- Examples:

-- Metal: $\gamma'(Ni_3Al)$-α(Mo)
by eutectic solidification.

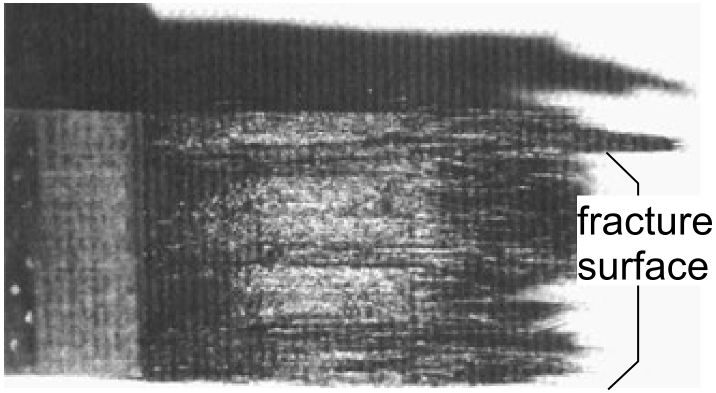
matrix: α(Mo) (ductile)



fibers: γ' (Ni_3Al) (brittle)

From W. Funk and E. Blank, “Creep deformation of Ni_3Al-Mo in-situ composites", *Metall. Trans. A* Vol. 19(4), pp. 987-998, 1988. Used with permission.

-- Ceramic: Glass w/SiC fibers
formed by glass slurry
E_{glass} = 76 GPa; E_{SiC} = 400 GPa.

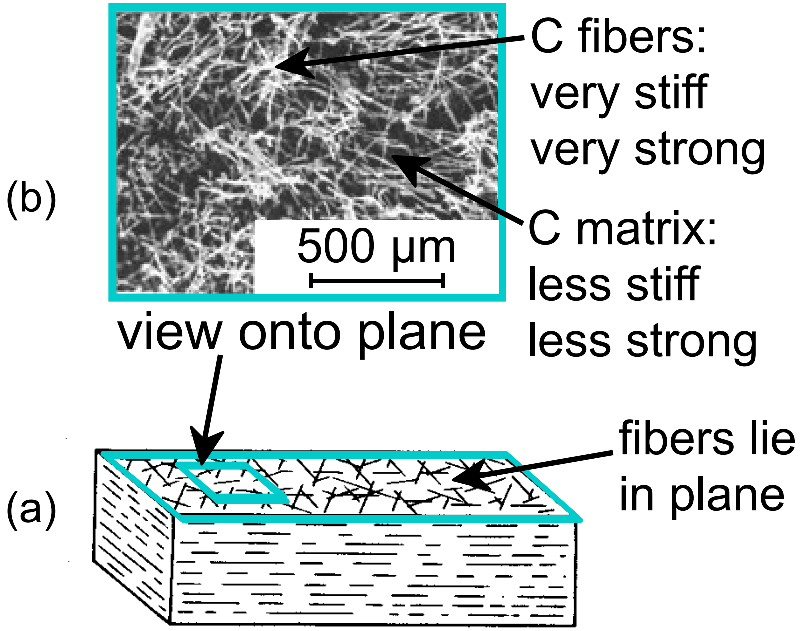


From F.L. Matthews and R.L. Rawlings, *Composite Materials; Engineering and Science*, Reprint ed., CRC Press, Boca Raton, FL, 2000. Used with permission of CRC Press, Boca Raton, FL.

Classification: Fiber-Reinforced (iv)

Particle-reinforced | **Fiber-reinforced** | Structural

- Discontinuous fibers, random in 2 dimensions
- Example: Carbon-Carbon
  - -- fabrication process:
    - - carbon fibers embedded in polymer resin matrix,
    - - polymer resin pyrolyzed at up to 2500° C.
  - -- uses: disk brakes, gas turbine exhaust flaps, missile nose cones.
- Other possibilities:
  - -- Discontinuous, random 3D
  - -- Discontinuous, aligned



Adapted from F.L. Matthews and R.L. Rawlings, *Composite Materials; Engineering and Science*, Reprint ed., CRC Press, Boca Raton, FL, 2000. (a) Fig. 4.24(a), p. 151; (b) Fig. 4.24(b) p. 151. (Courtesy I.J. Davies) Reproduced with permission of CRC Press, Boca Raton, FL.

Classification: Fiber-Reinforced (v)

Particle-reinforced | **Fiber-reinforced** | Structural

- Critical fiber length for effective stiffening & strengthening:

fiber ultimate tensile strength

fiber diameter

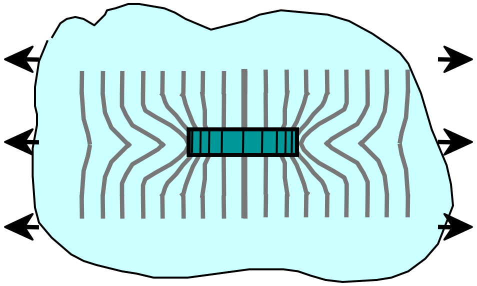
$$\text{fiber length} > \frac{\sigma_f d}{2\tau_c}$$

shear strength of
fiber-matrix interface

- Ex: For fiberglass, common fiber length > 15 mm needed
- For longer fibers, stress transference from matrix is more efficient

Short, thick fibers:

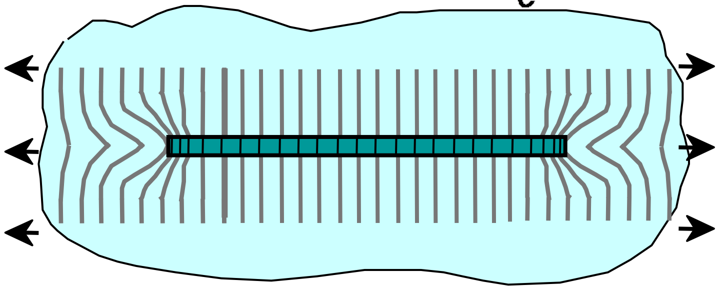
$$\text{fiber length} < \frac{\sigma_f d}{2\tau_c}$$



Low fiber efficiency

Long, thin fibers:

$$\text{fiber length} > \frac{\sigma_f d}{2\tau_c}$$



High fiber efficiency

Composite Stiffness: Longitudinal Loading

Continuous fibers - Estimate fiber-reinforced composite modulus of elasticity for continuous fibers

- Longitudinal deformation

$\sigma_c = \sigma_m V_m + \sigma_f V_f$ and $\varepsilon_c = \varepsilon_m = \varepsilon_f$

volume fraction

isostrain

$$\therefore \quad \boxed{E_{cl} = E_m V_m + E_f V_f}$$

E_{cl} = longitudinal modulus

c = *composite*

f = fiber

m = matrix

Composite Stiffness: Transverse Loading

- In transverse loading the fibers carry less of the load

$\varepsilon_c = \varepsilon_m V_m + \varepsilon_f V_f$ and $\sigma_c = \sigma_m = \sigma_f = \sigma$

$$\uparrow$$

isostress

$$\boxed{\frac{1}{E_{ct}} = \frac{V_m}{E_m} + \frac{V_f}{E_f}}$$

$$E_{ct} = \frac{E_m E_f}{V_m E_f + V_f E_m}$$

E_{ct} = transverse modulus

c = *composite*

f = fiber

m = matrix

Composite Stiffness

Particle-reinforced | **Fiber-reinforced** | Structural

- Estimate of E_{cd} for discontinuous fibers:

-- valid when fiber length < $15\dfrac{\sigma_f d}{\tau_c}$

-- Elastic modulus in fiber direction:

$$E_{cd} = E_m V_m + K E_f V_f$$

efficiency factor:

-- aligned: $K = 1$ (aligned parallel)
-- aligned: $K = 0$ (aligned perpendicular)
-- random 2D: $K = 3/8$ (2D isotropy)
-- random 3D: $K = 1/5$ (3D isotropy)

Table 16.3, *Callister & Rethwisch 10e*.
(Source is H. Krenchel, *Fibre Reinforcement*, Copenhagen: Akademisk Forlag, 1964.)

Composite Production Methods (i)

Pultrusion

- Continuous fibers pulled through resin tank to impregnate fibers with thermosetting resin
- Impregnated fibers pass through steel die that preforms to the desired shape
- Preformed stock passes through a curing die that is
  - precision machined to impart final shape
  - heated to initiate curing of the resin matrix

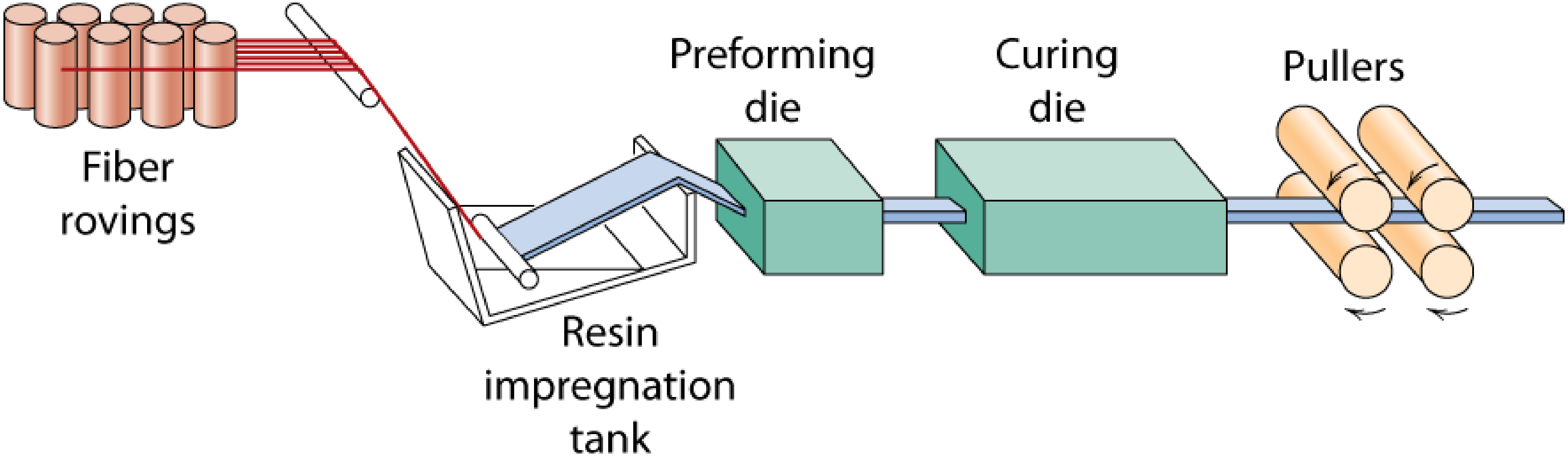


Fig. 16.13, *Callister & Rethwisch 10e*.

Composite Production Methods (ii)

- Filament Winding
  - Continuous reinforcing fibers are accurately positioned in a predetermined pattern to form a hollow (usually cylindrical) shape
  - Fibers are fed through a resin bath to impregnate with thermosetting resin
  - Impregnated fibers are continuously wound (typically automatically) onto a mandrel
  - After appropriate number of layers added, curing is carried out either in an oven or at room temperature
  - The mandrel is removed to give the final product

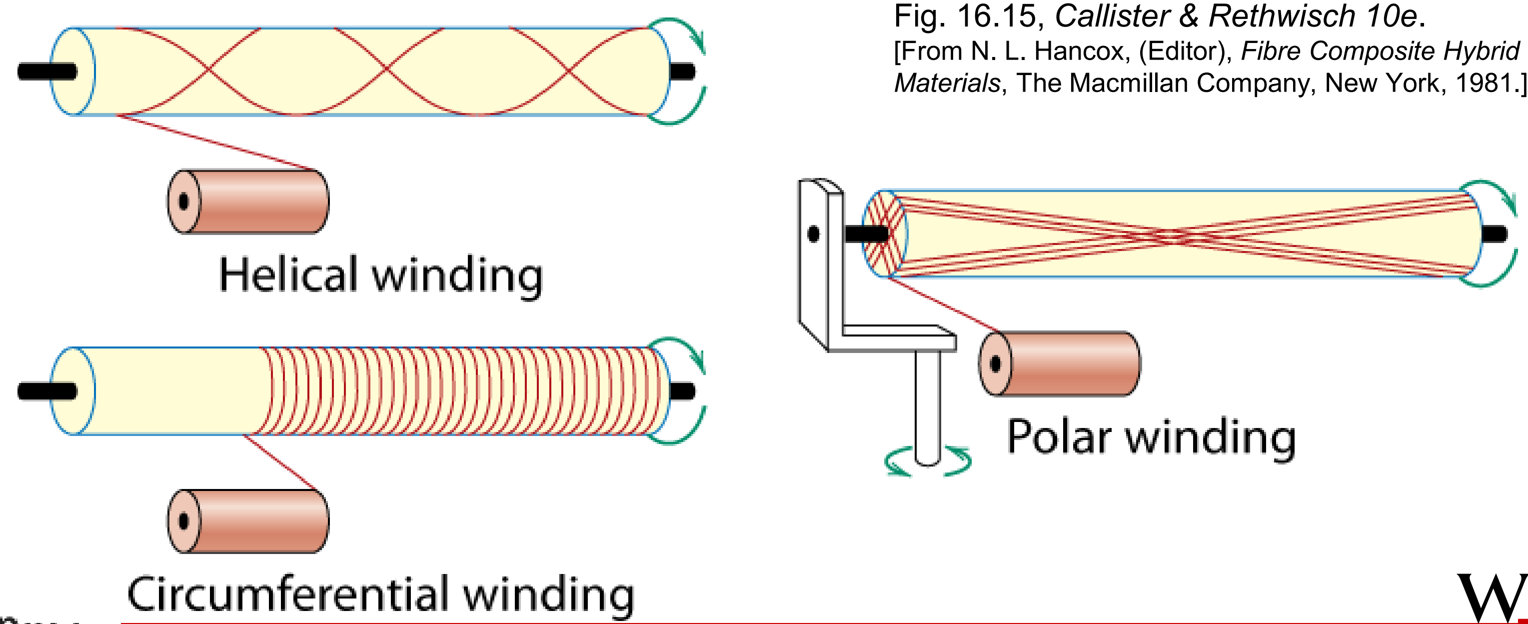


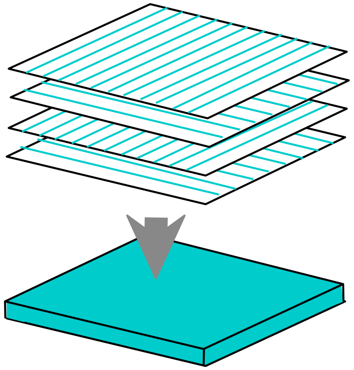
Fig. 16.15, *Callister & Rethwisch 10e*.
[From N. L. Hancox, (Editor), *Fibre Composite Hybrid Materials*, The Macmillan Company, New York, 1981.]

Classification: Structural

Particle-reinforced　Fiber-reinforced　**Structural**

- **Laminates** -

  -- stacked and bonded fiber-reinforced sheets
    - stacking sequence: e.g., 0º/90º
    - benefit: balanced in-plane stiffness



- **Sandwich panels**

  -- honeycomb core between two facing sheets
    - benefits: low density, large bending stiffness

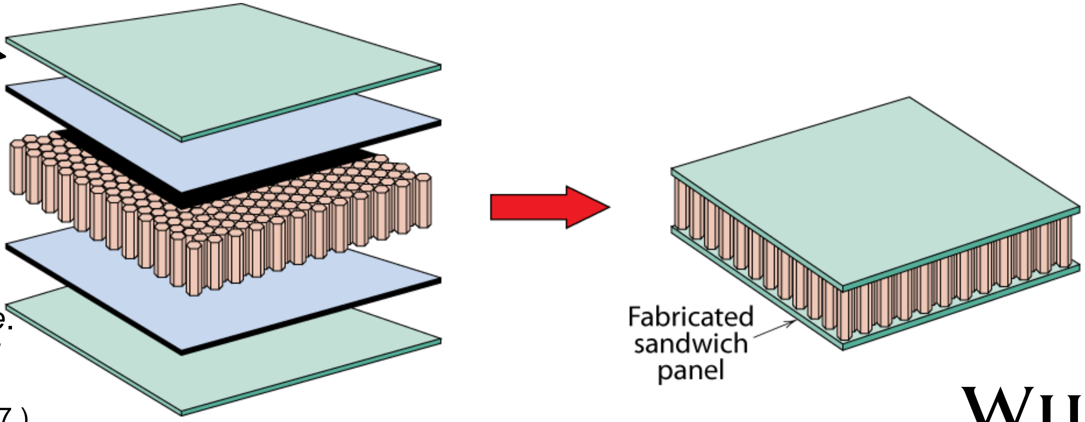
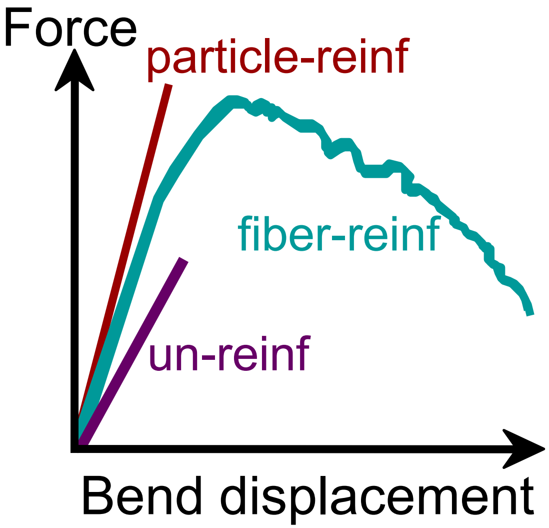


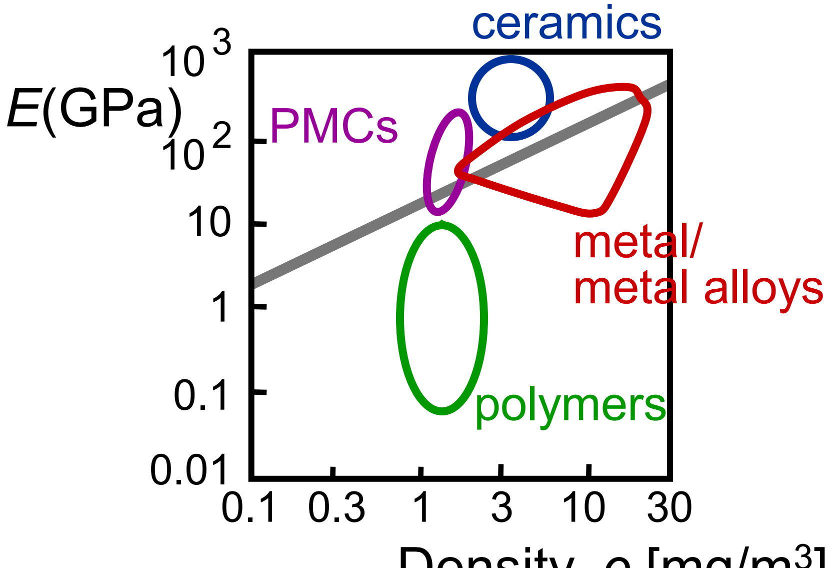
Fig. 16.18, *Callister & Rethwisch 10e*. (Reprinted with permission from *Engineered Materials Handbook*, Vol. 1, Composites, ASM International, Materials Park, OH, 1987.)

Composite Benefits

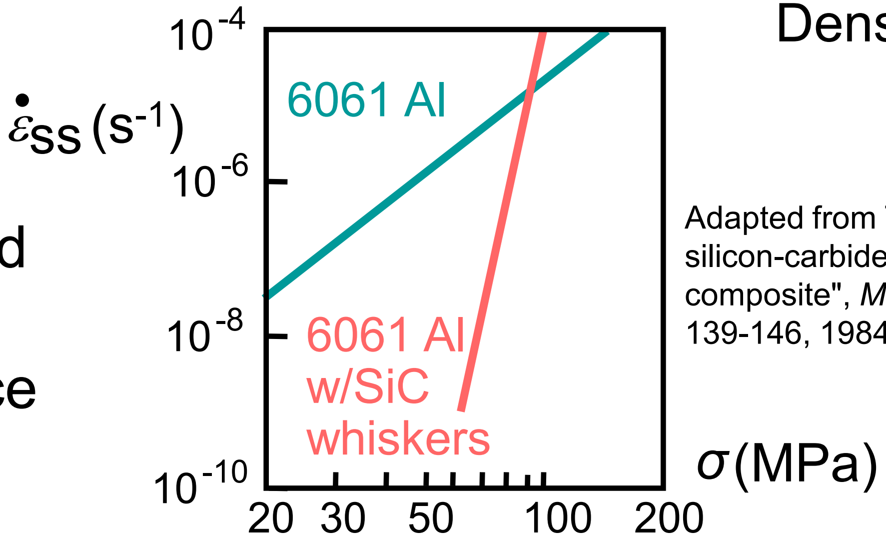
- CMCs: Increased toughness



- PMCs: Increased E/ρ



- MMCs:
  Increased creep resistance



Adapted from T.G. Nieh, "Creep rupture of a silicon-carbide reinforced aluminum composite", *Metall. Trans. A* Vol. 15(1), pp. 139-146, 1984. Used with permission.

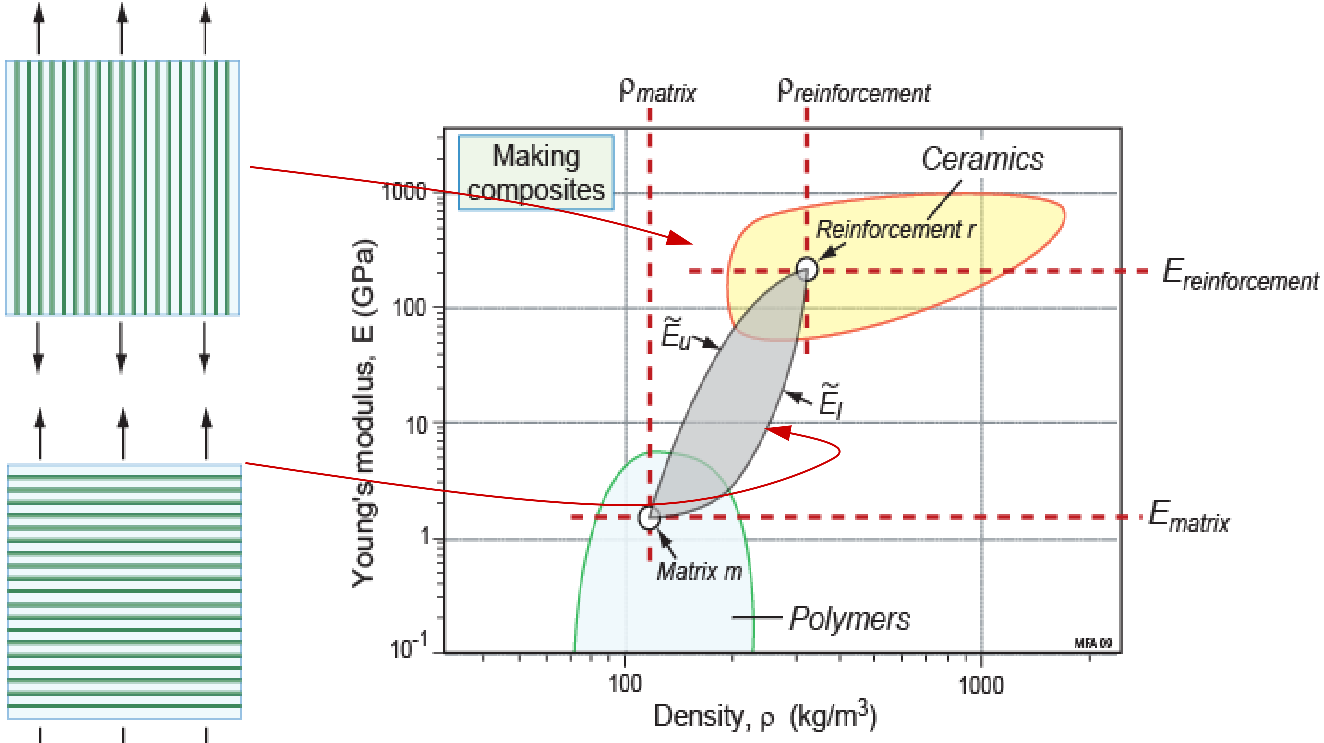
Control of modulus by architecture

Composites: “solid – solid hybrids”

CFRP

GFRP

Upper bound:



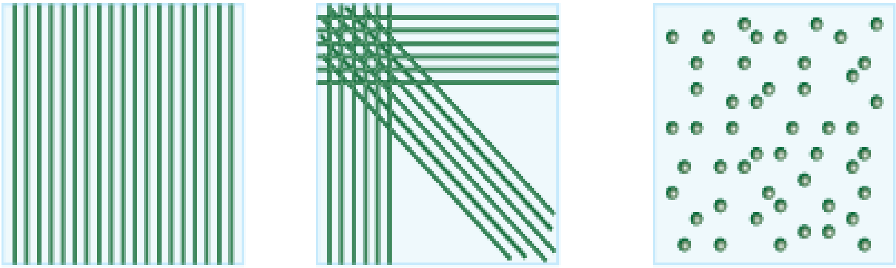
Lower bound: E_l

case study: control by architecture and size effects

Familiar Architectures

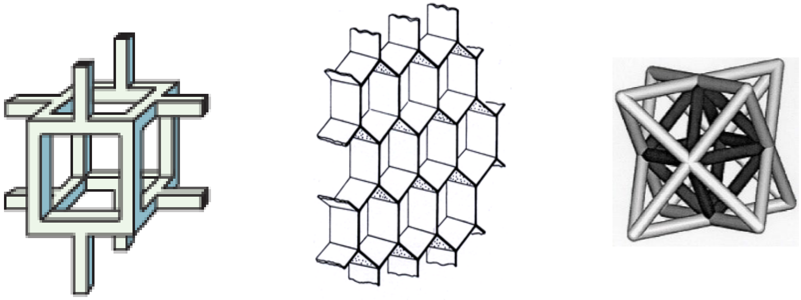
Composites

- *Unidirectional*
- *Quasi-isotropic*
- *Particulate*



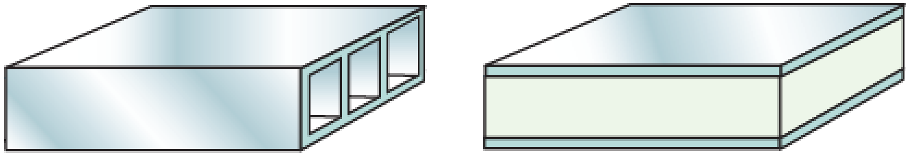
Cellular structures

- *Foams*
- *Honeycombs*
- *Triangulated lattices*



Sandwich structures

- *Symmetric sandwiches*



Many more

Summary

- Composites types are designated by:
  - the matrix material (CMC, MMC, PMC)
  - the reinforcement (particles, fibers, structural)
- Composite property benefits:
  - MMC: enhanced E, σ^*, creep performance
  - CMC: enhanced K_{Ic}
  - PMC: enhanced E/ρ, σ_y, TS/ρ
- Particulate-reinforced:
  - Types: large-particle and dispersion-strengthened
  - Properties are isotropic
- Fiber-reinforced:
  - Types: continuous (aligned)
discontinuous (aligned or random)
  - Properties can be isotropic or anisotropic
- Structural:
  - Laminates and sandwich panels